2003 STATE MATH CONTEST

Preliminary Exam - Grades 10-12

1. An equilateral triangle is inscribed inside a circle and second circle is inscribed inside the triangle. What is the ratio of the area of the second circle to the area of the original circle?

   (a) 1:2π (b) 1:4 (c) 1:6 (d) 1:π (e) none of these

2. Which is the inverse of the function $f(x) = \frac{1}{3}\ln\left(x + \sqrt{x^2+1}\right)$?

   (a) $3\left(e^{3x} + e^{-3x}\right)$ (b) $\frac{1}{3}\left(e^{3x} + e^{-3x}\right)$ (c) $\frac{1}{2}\left(e^{3x} + e^{-3x}\right)$

   (d) $\frac{1}{2}\left(e^{3x} - e^{-3x}\right)$ (e) none of these

3. Three positive whole numbers x, y, z form a geometric sequence with a sum of 42. If y is multiplied by 5/4, the numbers x, $\frac{5y}{4}$, z now form an arithmetic sequence. What is the largest possible value of x?

   (a) 24 (b) 6 (c) 28 (d) 30 (e) none of these

4. Which of the following three equation are identities?

   (i) $[\csc(x) - \cot(x)][1 + \cos(x)] = \sin(x)$

   (ii) $\sec(x)\csc(x) - \cot(x) = \tan(x)$

   (iii) $\dfrac{1 + \cos(2x)}{\sin(2x)\cos(x)} = \csc(x)$

   (a) only (i) (b) only (i) and (ii) (c) only (ii) and (iii)

   (d) only (i) and (iii) (e) all are identities

5. If a committee of 3 men and 3 women were randomly selected from a group of 9 men and 11 women, what would be the probability that a particular men and a particular women would both be on the committee?

   (a) 1/11 (b) 1/9 (c) 2/9 (d) 1/6 (e) 2/6

6. The equation $2x^4 - 3x^3 - 14x^2 - 22x - 8 = 0$ has two real and two complex solutions. What is the product of the two complex solutions?

   (a) -1 (b) 1 (c) 2 (d) 5 (e) none of these

7. A particle is moving on the x-axis with an acceleration of $A(t) = -4\pi^2 \cos(\pi t) - 6t - \dfrac{1}{(t+1)^2}$ where t is time. If its velocity at time zero is 4 and its position at time zero is 7, what is its position at time $t = 1$?

   (a) $7 + \ln 2$ (b) $5 + \ln 2$ (c) $2 + \ln 5$ (d) $2 + \ln 7$ (e) none of these

8. How many different 4-digit numbers are possible to construct using the digits $\{1, 3, 4, 6, 7, 8\}$ that satisfy all of these conditions:

   - the number is between 3300 and 7200
   - the number is even
   - the number has no repeated digit.

   (a) 72 (b) 84 (c) 96 (d) 102 (e) 108

9. The notation 231_p stands for a number written in base p and equals $2p^2 + 3p + 1$. Compute $(123_4)(212_4)$. Which is the answer in base 4?

(a) 100102_4 (b) 200102_4 (c) 100012_4 (d) 100002_4 (e) none of these

10. From a point A on a level plane, the angle of elevation to the top of a hill is $25°$. On the same level, but 60 foot closer, the angle of elevation is $35°$. Which of the following expressions gives the height of the hill?

(a) $60\tan(25°)$ (b) $\dfrac{60\sin(25°)\sin(35°)}{\sin(10°)}$ (c) $\dfrac{60(\tan(35°) - \tan(25°))}{\tan(25°)\tan(35°)}$

(d) $\dfrac{60\tan(25°)\tan(25°)}{\tan(10°)}$ (e) none of these

11. For positive real numbers x and y define $x \star y = \dfrac{x+y}{xy}$. Which of the following are true for all positive numbers x, y and z ?

(i) $1 \star x = x$ (iii) $(x \star y) \star z = x \star (y \star z)$

(ii) $x \star y = y \star x$ (iv) $(x + y) \star z = x \star z + y \star z$

(a) all are true (b) none are true (c) only (ii) is true

(d) only (i) and (ii) are true (e) only (ii) and (iii) are true

12. If two fair, standard dice are thrown, which one of the following is the closest to the probability that either the total on the two will be seven or at least one of them will show a four?

(a) 0.25 (b) 0.30 (c) 0.35 (d) 0.40 (e) 0.45

13. If m is the minimum value attained by $f(x, y) = x^2 + y^2 - 10x + 6y + 27$ then

(a) $-15 < m < -12$ (b) $-12 < m < -9$ (c) $-9 < m < -6$
(d) $-6 < m < -3$ (e) $-3 < m < 0$

14. A circle with center at $(15, -3)$ is tangent to $y = \frac{1}{3}x^2$ at a point in the first quadrant. The radius of that circle is equal to:

(a) $5\sqrt{6}$ (b) $8\sqrt{3}$ (c) $9\sqrt{2}$ (d) $5\sqrt{7}$ (e) $6\sqrt{5}$

15. A goat is tethered on a 16 foot rope tied to the corner of a square shed that has sides of 12 feet. What is the maximum grazing area for the goat?

(a) 200π ft^2 (b) 192π ft^2 (c) 112π ft^2 (d) 196π ft^2 (e) none of these

16. Adam, Bonnie, Connie, Dan and Elly went to high school together, although Adam dropped out before graduation. Eventually their professions became actuary, base fiddle player, computer programmer, clothes designer, and electrical engineer, although not necessarily respectively. Connie, and the engineer, and the actuary, frequently play bridge together. Elly joins them sometimes after game to go to movies. The base player left the state and never had contact with any of the others again. Dan, who knows nothing about music dates the computer programmer. The clothes designer, who doesn't play cards, is a widow. The electrical engineer is married to her college sweetheart. Who is the engineer?

(a) Adam (b) Bonnie (c) Connie (d) Dan (e) Elly