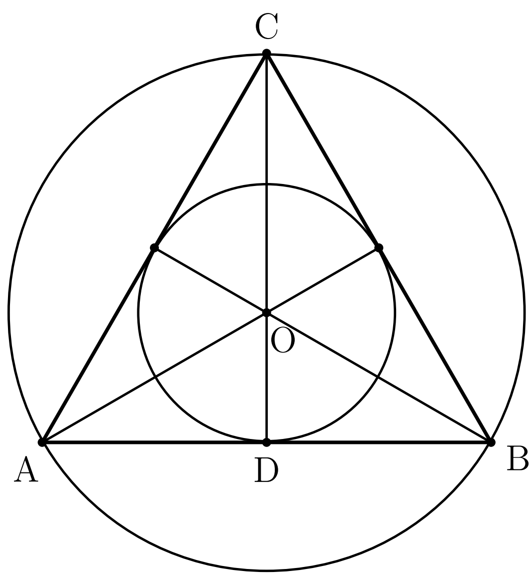
Utah State Math Contest
Preliminary Exam with Solutions, Grades 10-12.

Problem 1.
An equilateral triangle is inscribed inside a circle and second circle is inscribed inside the triangle. What is the ratio of the area of the second circle to the area of the original circle?

(a) 1:2π (b) 1:4 (c) 1:6 (d) 1:π (e) none of these

Note. *In an equilateral triangle, angle bisectors, perpendicular bisectors and medians all coincide and the point the their intersection is the geometric center of the triangle, which is the common center of both the inscribed and circumscribed circle.*



Solution 1. Denote the vertices of the equilateral triangle by A, B, and C. Let O be the common center of both circles, and D be the point of tangency of the inscribed circle with side $\overline{AB}$ of the triangle. Then $\triangle AOD$ is a right triangle with angle at vertex A of 30°. In this triangle the hypothenuse AO is R and side OD is r, where R and r are radii of the larger and smaller circles respectively. Therefore

$$\frac{r}{R} = \sin 30^\circ = \frac{1}{2}. \qquad \frac{\pi r^2}{\pi R^2} = \frac{1}{4}.$$

The desired ratio of areas is 1:4.

Solution 2. We know that the medians divide each other in a ratio of $1:2$. Consider the median $\overline{CD}$. The smaller part of this median, the line segment $\overline{OD}$, is exactly the radius of the smaller circle, and the larger part, the line segment $\overline{OC}$, is the radius of the larger circle. Therefore the ratio of radii is $1:2$ and the ratio of areas is $1:4$.

Problem 2.
Which is the inverse of the function $f(x) = \dfrac{1}{3}\ln\left(x + \sqrt{x^2+1}\right)$?

(a) $3\left(e^{3x} + e^{-3x}\right)$ (b) $\dfrac{1}{3}\left(e^{3x} + e^{-3x}\right)$ (c) $\dfrac{1}{2}\left(e^{3x} + e^{-3x}\right)$
(d) $\dfrac{1}{2}\left(e^{3x} - e^{-3x}\right)$ (e) none of these

Solution. To switch the input and output of the function first interchange x and y in the function's formula $y = \dfrac{1}{3}\ln\left(x + \sqrt{x^2+1}\right)$, and then solve for y.

$$\begin{aligned} x &= \frac{1}{3}\ln\left(y + \sqrt{y^2+1}\right) \\ e^{3x} &= y + \sqrt{y^2+1} \end{aligned} \tag{1}$$

To solve for y we can now proceed in two ways.

$$\begin{aligned} \left(e^{3x} - y\right)^2 &= y^2 + 1 \\ e^{6x} - 2ye^{3x} + y^2 &= y^2 + 1 \\ 2ye^{3x} &= e^{6x} - 1 \\ y = \frac{e^{6x}-1}{2e^{3x}} &= \frac{e^{3x} - e^{-3x}}{2} \end{aligned}$$

To proceed differently, the trick is to eliminate the square root from the equation (1) without squaring it by taking reciprocals of both sides.

$$e^{-3x} = \frac{1}{y + \sqrt{y^2+1}} = \frac{1}{y + \sqrt{y^2+1}}\,\frac{y - \sqrt{y^2+1}}{y - \sqrt{y^2+1}} = \frac{y - \sqrt{y^2+1}}{y^2 - (y^2+1)} = -y + \sqrt{y^2+1} \tag{2}$$

Combining (1) and (2) gives $y = \dfrac{e^{3x} - e^{-3x}}{2}$.

Problem 3.
The first three terms of a geometric sequence are x, y, z and these have a sum of 42. If the middle term y is multiplied by 5/4, the numbers $x, \frac{5y}{4}, z$ now form an arithmetic sequence. What is the largest possible value of x?

(a) 24 (b) 6 (c) 28 (d) 30 (e) none of these

Solution 1. In a geometric sequence, the ratio of any term to the proceeding one is constant. That gives the first of the following equations. In an arithmetic sequence, a similar statement is true: the difference of any term and the proceeding one is constant. That gives the second equation.

$$\left\{\begin{array}{rcl} \frac{y}{x} &=& \frac{z}{y} \\ \frac{5y}{4} - x &=& z - \frac{5y}{4} \\ x + y + z &=& 42 \end{array}\right. \quad \text{yields} \quad \left\{\begin{array}{rcl} y^2 &=& xz \\ \frac{10y}{4} &=& (x+z) \\ (x+z) &=& 42 - y \end{array}\right. \quad \text{and} \quad \left\{\begin{array}{rcl} y^2 &=& xz \\ \frac{10y}{4} &=& (x+z) \\ \frac{10y}{4} + y &=& 42 \end{array}\right. .$$

Solving the last equation for y, etc., leads to $(x = 24,\ y = 12,\ z = 6)$ or $(x = 6,\ y = 12,\ z = 24)$. The largest possible value of x is 24.

Solution 2. The three terms of the geometric sequence with the common ratio r and first term x are $x,\ xr,\ xr^2$. After multiplying the middle term by 5/4 we'll get an arithmetic sequence with a common difference. This yields $\dfrac{5}{4}xr - x = xr^2 - \dfrac{5}{4}xr$. After cancelling by x we obtain a quadratic equation $2r^2 - 5r + 2 = 0$, with roots $r = \frac{1}{2}$ and $r = 2$.
Substituting these in $x + xr + xr^2 = 42$ we get $x = 6$ or 24.

Problem 4.
Which of the following three equation are identities?
(i) $[\csc(x) - \cot(x)][1 + \cos(x)] = \sin(x)$
(ii) $\sec(x)\csc(x) - \cot(x) = \tan(x)$
(iii) $\dfrac{1 + \cos(2x)}{\sin(2x)\cos(x)} = \csc(x)$

(a) only (i) (b) only (i) and (ii) (c) only (ii) and (iii)
(d) only (i) and (iii) (e) all are identities

Solution. To prove an identity, a standard method is to start with its left hand side and via simpler identities arrive at the right-hand side.

(i) $LHS = [\csc(x) - \cot(x)][1 + \cos(x)] = \left[\dfrac{1}{\sin(x)} - \dfrac{\cos(x)}{\sin(x)}\right][1 + \cos(x)] =$
$\left[\dfrac{1 - \cos(x)}{\sin(x)}\right][1 + \cos(x)] = \left[\dfrac{1 - \cos^2(x)}{\sin(x)}\right] = \left[\dfrac{\sin^2(x)}{\sin(x)}\right] = \sin(x) = RHS$

(ii) $LHS = \sec(x)\csc(x) - \cot(x) = \left[\dfrac{1}{\cos(x)}\dfrac{1}{\sin(x)} - \dfrac{\cos(x)}{\sin(x)}\right] =$
$\dfrac{1}{\sin(x)}\left[\dfrac{1}{\cos(x)} - \cos(x)\right] = \dfrac{1}{\sin(x)}\left[\dfrac{1 - \cos^2(x)}{\cos(x)}\right] = \dfrac{\sin(x)}{\cos(x)} = \tan(x) = RHS$

(iii) $LHS = \dfrac{1 + \cos(2x)}{\sin(2x)\cos(x)} = \dfrac{1 + [2\cos^2(x) - 1]}{[2\sin(x)\cos(x)]\cos(x)} = \dfrac{1}{\sin(x)} = \csc(x) = RHS$

All three equations are identities.

Problem 5.
If a committee of 3 men and 3 women was randomly selected from a group of 9 men and 11 women, what would be the probability that a particular man and a particular woman would both be on the committee?

(a) 1/11 (b) 1/9 (c) 2/9 (d) 1/6 (e) 2/6

Solution. The order of choosing the committee members is irrelevant, so we will use combinations, not permutations. If a particular woman is on the committee, then the two remaining women are chosen from the ten other women. It can be done in $\mathrm{C}(10, 2) = \binom{10}{2}$ number of ways. Similarly, there are $\mathrm{C}(8, 2) = \binom{8}{2}$ ways to choose three men and guarantee that a particular man is on. Taking the ratio of the product of these numbers over total number of committees with any three men and any three women out of nine and eleven, respectively, gives

$$\frac{\mathrm{C}(8,2)\mathrm{C}(10,2)}{\mathrm{C}(9,3)\mathrm{C}(11,3)} = \frac{\frac{8\cdot7}{2}\cdot\frac{10\cdot9}{2}}{\frac{9\cdot8\cdot7}{3\cdot2}\cdot\frac{11\cdot10\cdot9}{3\cdot2}} = \frac{3\cdot3}{9\cdot11} = \frac{1}{11}\ .$$

Problem 6.
The equation $2x^4-3x^3-14x^2-22x-8=0$ has two real and two complex solutions. What is the product of the two complex solutions?

(a) -1 (b) 1 (c) 2 (d) 5 (e) none of these

Solution. If a rational number $\frac{p}{q}$ (in lowest terms) is a solution of the equation $P(x)=0$, where $P(x)=2x^4-3x^3-14x^2-22x-8$, then the integer p divides -8 and the integer q divides 2. Therefore any rational root of the equation must be among the numbers

$$\pm\frac{1}{2},\ \pm 1,\ \pm 2,\ \pm 4,\ \pm 8.$$

By substituting we find that $P\left(-\frac{1}{2}\right)=0$ and $P(4)=0$. In other words, $x_1=-1/2$ and $x_2=4$ are the real (and rational) solutions of the equation $P(x)=0$. Now we can proceed in two different ways.

(A) It is not necessary to find the other roots numerically to solve the problem. Let x_3 and x_4 be the other (complex) roots. Writing $P(x)$ as a product of factors and comparing coefficients of the free term (a coefficient of x^0) we will get a condition for the product of all the roots:

$$P(x)=2x^4-3x^3-14x^2-22x-8=2(x-x_1)(x-x_2)(x-x_3)(x-x_4)=$$

$$2x^4+(?)x^3+(?)x^2+(?)x+2x_1x_2x_3x_4.$$

Therefore

$$x_3x_4=\frac{-8}{2x_1x_2}=\frac{-8}{2\cdot(-0.5)\cdot 4}=2.$$

The product of the complex solutions is 2.

(B) Since $P(x)$ has integer coefficients, x_3 and x_4 are complex conjugates and their product is a real number. To find x_3 and x_4 divide $P(x)$ by $(2x+1)(x-4)$ to get x^2+2x+2. Now solve $x^2+2x+2=0$ by using either the quadratic formula or completing the square:

$$x=\frac{-2\pm\sqrt{2^2-4\cdot 2}}{2}=-1\pm i \qquad \text{or} \qquad (x+1)^2=-1,\quad x+1=\pm i;\quad x=-1\pm i.$$

This yields a product of $(-1+i)(-1-i)=2$.

Problem 7.
A particle is moving on the x-axis with an acceleration of $A(t)=-4\pi^2\cos(\pi t)-6t-\frac{1}{(t+1)^2}$ where t is time. If its velocity at time zero is 4 and its position at time zero is 7, what is its position at time $t=1$?

(a) $7+\ln 2$ (b) $5+\ln 2$ (c) $2+\ln 5$ (d) $2+\ln 7$ (e) none of these

Solution. Let $S(t)$ be the position, $V(t)$ be the velocity, and $A(t)$ be the acceleration of the particle at time t.
We have: $S'(t)=V(t),\ S''(t)=V'(t)=A(t),\ S(0)=7,\ V(0)=4.$

$$V(t)=\int A(t)dt=\int\left(-4\pi^2\cos(\pi t)-6t-\frac{1}{(t+1)^2}\right)dt=-4\pi\sin(\pi t)-3t^2+\frac{1}{t+1}+C_1$$

Using the initial condition $V(0)=4$ yields $C_1=3$. Similarly,

$$S(t)=\int V(t)dt=\int\left(-4\pi\sin(\pi t)-3t^2+\frac{1}{t+1}+3\right)=4\cos(\pi t)-t^3+\ln(1+t)+3t+C_2.$$

Using the initial condition $S(0)=7$ yields $C_2=3$. Thus $S(1)=-4-1+\ln 2+3+3=1+\ln 2$.

Problem 8.

How many different 4-digit numbers are possible to construct using the digits $\{1, 3, 4, 6, 7, 8\}$ if those 4-digit numbers must satisfy all of these conditions:
- the number is between 3300 and 7200
- the number is even
- the number has no repeated digit.

(a) 72 (b) 84 (c) 96 (d) 102 (e) 108

Solution. Let $xyzw$ be such a four digit number. Then either $x = 3$, $x = 4$, $x = 6$ or $x = 7$.
If $x = 4$ then there are 2 choices for w: 6 and 8, that guarantee the number is even. We can choose any of the remaining numbers for y. This gives $6 - 2 = 4$ possible choices for y and $6 - 3 = 3$ choices for z. Therefore there are $1 \cdot 4 \cdot 3 \cdot 2 = 24$ different four digit, even numbers that start with 4.
The case $x = 6$ is similar, giving another 24 solutions.
If $x = 3$ then w is one of $\{4, 6, 8\}$. Now y must be greater than 3, but one of the even numbers has already been used leaving three choices. Now z can be any of the remaining three numbers, resulting in $1 \cdot 3 \cdot 3 \cdot 3 = 27$ choices.
Finally, if $x = 7$, then $y = 1$ and the number of choices is $1 \cdot 1 \cdot (6 - 3) \cdot 3 = 9$.
The total number of choices is $27 + 24 + 24 + 9 = 84$.

Problem 9.

The notation 231_p stands for a number written in base p and equals $2p^2 + 3p + 1$. Compute $(123_4)(212_4)$. Which is the answer in base 4?

(a) 100102_4 (b) 200102_4 (c) 100012_4 (d) 100002_4 (e) none of these

Solution 1. In base 10 every time two numbers add to ten we must carry one to the next place value. In base p, every time two numbers add to p we must carry one to the next place value.
For example $3 \cdot 4^2 + 2 \cdot 4^2 = 5 \cdot 4^2 = (4 + 1)4^2 = 4^3 + 4^2$. Thus

$$\begin{array}{cccccc}
 & & & 1 & 2 & 3_4 \\
 & & & 2 & 1 & 2_4 \\
\cline{4-6}
 & & & 3 & 1 & 2_4 \\
 & & 1 & 2 & 3_4 & \\
 & 3 & 1 & 2_4 & & \\
\hline
1 & 0 & 0 & 0 & 0 & 2_4
\end{array}$$

Solution 2. We will perform the above calculations using standard notation. Sometimes it is easier to start with the general case of base p.

$$(123_p)(212_p) = (p^2 + 2p + 3)(2p^2 + p + 2) = 2p^4 + 5p^3 + 10p^2 + 7p + 6.$$

In our case, $p = 4$ and all coefficients should be less than 4.

$$2p^4 + 5p^3 + 10p^2 + 7p + 6 = 2 \cdot 4^4 + (4 + 1)4^3 + (2 \cdot 4 + 2)4^2 + (4 + 3)4 + 4 + 2 =$$

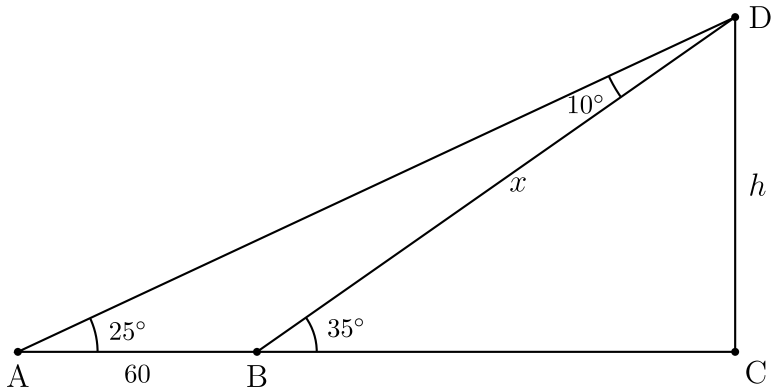
$$= 3 \cdot 4^4 + 3 \cdot 4^3 + 3 \cdot 4^2 + 4 \cdot 4 + 2 = 3 \cdot 4^4 + 3 \cdot 4^3 + 4 \cdot 4^2 + 2 = 3 \cdot 4^4 + 4 \cdot 4^3 + 2 = 4 \cdot 4^4 + 2 = 4^5 + 2 =$$

$$= 1 \cdot 4^5 + 0 \cdot 4^4 + 0 \cdot 4^3 + 0 \cdot 4^2 + 0 \cdot 4^1 + 2 \cdot 4^0 = 100002_4.$$

We really didn't need a calculator ...

Problem 10.
From a point A on a level plane, the angle of elevation to the top of a hill is 25°. On the same level, but 60 foot closer, the angle of elevation is 35°. Which of the following expressions gives the height of the hill?

(a) $60\tan(25°)$ (b) $\dfrac{60\sin(25°)\sin(35°)}{\sin(10°)}$ (c) $\dfrac{60(\tan(35°)-\tan(25°))}{\tan(25°)\tan(35°)}$
(d) $\dfrac{60\tan(25°)\tan(25°)}{\tan(10°)}$ (e) none of these



Solution. Using the notation of the figure above, $\sphericalangle ADB = 10°$. In $\triangle BCD$ $\sin(35°) = \dfrac{h}{x}$ so $h = x\sin(35°)$.
Using the law of sines in $\triangle ABD$ gives $\dfrac{x}{\sin(25°)} = \dfrac{60}{\sin(10°)}$ so $x = \dfrac{60\sin(25°)}{\sin(10°)}$.
Substituting yields $h = \dfrac{60\sin(25°)\sin(35°)}{\sin(10°)}$.

Problem 11.
For positive real numbers x and y define $x \star y = \dfrac{x+y}{xy}$. Which of the following are true for all positive numbers x, y and z ?

(i) $1 \star x = x$ (iii) $(x \star y) \star z = x \star (y \star z)$
(ii) $x \star y = y \star x$ (iv) $(x+y) \star z = x \star z + y \star z$

(a) all are true (b) none are true (c) only (ii) is true
(d) only (i) and (ii) are true (e) only (ii) and (iii) are true

Solution. To check if the listed properties hold, we use the definition of the $\star$ operation several times. It may be a useful trick for awhile to rewrite the definition using symbols that do not appear in the listed properties, for example a and b. Let us also recall that to disprove an identity it is enough to find one (numerical) counterexample. The proof of an identity, however, must be done in general.
Below, the LHS and RHS stand for the left and right hand side, respectively.

(i) $LHS = 1 \star x = \dfrac{1+x}{x} \neq x = RHS$. For example (i) doesn't hold if $x = 1$ because $2 \neq 1$.

(ii) $LHS = x \star y = \dfrac{x+y}{xy} = \dfrac{y+x}{yx} = y \star x = RHS$.
The middle equality holds because both addition and multiplication are commutative.

(iii) $LHS = (x \star y) \star z = \dfrac{x+y}{xy} \star z = \dfrac{\frac{x+y}{xy} + z}{\frac{x+y}{xy} \cdot z} = \dfrac{x+y+xyz}{(x+y)z}$

$RHS = x \star (y \star z) = x \star \frac{y+z}{yz} = \dfrac{x + \frac{y+z}{yz}}{x \cdot \frac{y+z}{yz}} = \dfrac{xyz+y+z}{x(y+z)}$

If $x = 1,\ y = 2,\ z = 3$ the $LHS = 1$ and $RHS = 11/5$, so $LHS \neq RHS$.

(iv) $LHS = (x+y) \star z = \dfrac{(x+y)+z}{(x+y)z}$ $\qquad RHS = x \star z + y \star z = \dfrac{x+z}{xz} + \dfrac{y+z}{yz}$

If $x = 1,\ y = 1,\ z = 1$ the $LHS = 3/2$ and $RHS = 4$, so $LHS \neq RHS$.

Only (ii) is true.

Note. *Often the properties of both multiplication and addition of real numbers are either unnoticed or taken for granted. Operation $\star$ is commutative, but it is not associative and unlike "usual" multiplication, the number 1 is not an identity element for $\star$.*

Problem 12.

If two fair, standard dice are thrown, which one of the following is the closest to the probability that either the total on the two will be seven or at least one of them will show a four?

(a) 0.25 (b) 0.30 (c) 0.35 (d) 0.40 (e) 0.45

Introduction. *When listing all possible outcomes, the most convenient method is to use ordered pairs. For example, $(3,5)$ means that 3 is obtained on the first die (say, a green die) and 5 is obtained on the second die (say, red). One needs ordered pairs because the result of obtaining 3 and 5 (without specifying on which die) is twice as likely as obtaining, say, a double 4. Calculating probabilities by counting requires outcomes that are equally likely.*

Solution 1. List all 36 possible outcomes, and then mark the ones having a sum of 7 or a 4 on at least one of the dice. There will be 15 of the possible 36 ordered pairs marked, so the answer is 15/36.

Solution 2. Let S be the event of obtaining a sum of 7 and F the event of getting one or more 4's. Notice that $(3,4)$ and $(4,3)$ are the only possibilities for S and F occurring simultaneously. Therefore $S \cap F$ happens with a probability of 2/36. Similarly, $P(S) = 6/36$ and $P(F) = 11/36$. Finally, S or F happens with a probability of 15/36 because

$$P(S \cup F) = P(S) + P(F) - P(S \cap F) = \frac{6}{36} + \frac{11}{36} - \frac{2}{36} = \frac{15}{36}.$$

Problem 13.

If m is the minimum value attained by $f(x,y) = x^2 + y^2 - 10x + 6y + 27$ then

(a) $-15 < m < -12$ (b) $-12 < m < -9$ (c) $-9 < m < -6$
(d) $-6 < m < -3$ (e) $-3 < m < 0$

Solution 1. Completing the squares is the easiest technique.

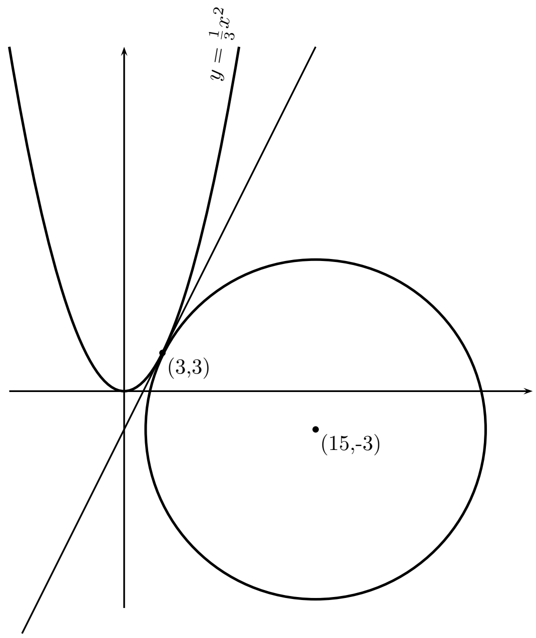
$$f(x,y) = x^2 + y^2 - 10x + 6y + 27 = (x-5)^2 + (y+3)^2 - 25 - 9 + 27 = (x-5)^2 + (y+3)^2 - 7.$$

The smallest possible output of -7 is obtained when the first two terms are zero (i.e. $x = 5$ and $y = -3$). Thus $-9 < m < -6$.

Solution 2. By partial differentiation one can justify that the minimum of the function is obtained at $(x,y) = (-5,-3)$ and is -7.

Problem 14.
A circle with center at $(15, -3)$ is tangent to $y = \frac{1}{3}x^2$ at a point in the first quadrant. The radius of that circle is equal to:

(a) $5\sqrt{6}$ (b) $8\sqrt{3}$ (c) $9\sqrt{2}$ (d) $5\sqrt{7}$ (e) $6\sqrt{5}$



Solution 1. The point of tangency minimizes the distance between the center of the circle and the points of the parabola. It is more convenient to work with the square of the distance.

$$\begin{cases} d^2(x,y) = (x-15)^2 + (y+3)^2 \\ y = \frac{1}{3}x^2 \end{cases}$$

Therefore we need to find a minimum of $d^2(x) = (x-15)^2 + \left(\dfrac{1}{3}x^2 + 3\right)^2$. The derivative of this function simplifies to $\dfrac{2}{9}(x-3)(2x^2+6x+45)$ and is equal to zero only when $x = 3$. Standard techniques show that there is a minimum at that point. Therefore $(3,3)$ is the point of tangency and the radius of the circle is $R = 6\sqrt{5}$.
Solution 2. The tangent line to both curves must be perpendicular to the radius, which has slope $\dfrac{y+3}{x-15}$, where (x,y) is the point of tangency. Slopes of perpendicular lines are negative reciprocals of one other and the tangent line to the parabola at point (x,y) has a slope of $\dfrac{2}{3}x$. Thus $\dfrac{2}{3}x = -\dfrac{x-15}{y+3}$.

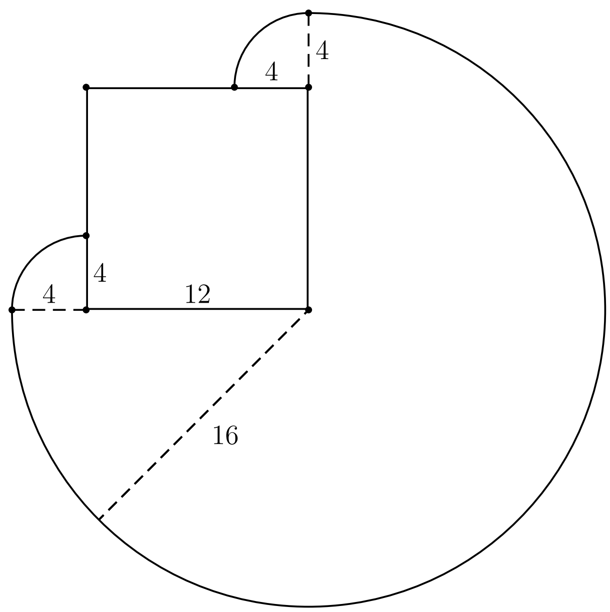
$$\begin{cases} y = \frac{1}{3}x^2 \\ \frac{2}{3}x(y+3) = 15 - x \end{cases} \qquad \text{substituting} \qquad \begin{cases} y = \frac{1}{3}x^2 \\ \frac{2}{3}x(\frac{1}{3}x^2 + 3) = 15 - x \end{cases}$$

The second equation leads to $2x^3 + 27x - 15 \cdot 9 = 0$. By considering the possible rational roots we find that $x = 3$ is a solution (and one can confirm that, as expected, it is the only solution). Then $y = 3$ and $R = 6\sqrt{5}$.
Solution 3. To find the coordinates (x, y) of the point of tangency consider simultaneously the equations of the circle and of the parabola. That gives two equations with three unknowns x, y, R- the radius. To solve the problem a third equation is needed. It can be obtained by calculating the slope of the common tangent line in two different ways: as a tangent to the parabola and as a tangent to the upper semicircle. (The circle is not a graph of a function, but the upper semicircle is.) After some tedious calculation this leads to the same system obtained in solution 2.

Problem 15.
A goat is tethered on a 16 foot rope tied to the corner of a square shed that has sides of 12 feet. What is the maximum grazing area for the goat?

(a) $200\pi\text{ft}^2$ (b) $192\pi\text{ft}^2$ (c) $112\pi\text{ft}^2$ (d) $196\pi\text{ft}^2$ (e) none of these

Solution.



We need to add 3/4 of the area of the big circle (with radius 16) and twice 1/4 of the area of the small circle (with radius 4). That yields

$$\frac{3}{4}\cdot 16\pi^2 + 2\cdot\frac{1}{4}\pi 4^2 = 200\pi.$$

Problem 16.
Adam, Bonnie, Connie, Dan and Elly went to high school together, although Adam dropped out before graduation. Eventually their professions became actuary, base fiddle player, computer programmer, clothes designer, and electrical engineer, although not necessarily respectively. Connie, and the engineer, and the actuary, frequently play bridge together. Sometimes Elly joins them after the game to go to the movies. The base player left the state and never had contact with any of the others again. Dan, who knows nothing about music, dates the computer programmer. The clothes designer, who doesn't play cards, is a widow. The electrical engineer is married to her college sweetheart. Who is the engineer?

(a) Adam (b) Bonnie (c) Connie (d) Dan (e) Elly

Solution. Since the electrical engineer is a woman, we will concentrate on Bonnie, Connie and Elly. The electrical engineer plays bridge together with Connie, so Connie is not the engineer. Elly isn't either because sometimes she joins the whole bridge party to go to the movies. It implies Bonnie is the electrical engineer.

Note. Connie plays bridge also with the actuary. She cannot be the base fiddle player either, because the musician doesn't keep in touch with the others, in particular doesn't play bridge with his/her classmates. Connie cannot be the clothes designer, because that person doesn't play cards. Thus Connie must be a computer programmer (and she is dating Dan.) The clothes designer as a widow is a woman, so it is Elly. Dan doesn't like music, so he is not a base fiddler player and that leaves him with the actuarial profession. Adam must be the base fiddle player, which doesn't necessarily require a high school diploma.