2003 STATE MATH CONTEST

Grades 7-9

1. The perimeters of a square and an equilateral triangle are equal. What is the ratio of the area of the triangle to the area of the square?

   (a) $\frac{4\sqrt{3}}{9}$ (b) $\sqrt{3}$ (c) $\frac{2\sqrt{3}}{3}$ (d) $\frac{8\sqrt{3}}{9}$ (e) none of these

2. Given that $x + y = 1$ and $x^3 + y^3 = \frac{49}{4}$, find the value of $x^2 + y^2$.

   (a) 15/2 (b) 17/2 (c) 19/2 (d) 21/2 (e)23/2

3. Compute $\sqrt{5 + \sqrt{21}} - \sqrt{5 - \sqrt{21}}$.

   (a) $\sqrt{5}$ (b) $\sqrt{6}$ (c) $\sqrt{7}$ (d) $\sqrt{8}$ (e) $\sqrt{10}$

4. Arthur's wife picks him up at the train station and drives him home every Friday. One Friday Arthur catches the early train, arrives 90 minutes early, and starts walking home. His wife, who left home at the usual time to pick him up, meets him on the way home. They arrive home 20 minutes earlier than normal. How many minutes had Arthur been walking before his wife picked him up?

   (a) 50 (b) 60 (c) 70 (d) 80 (e) 90

5. What is the number of elements in the smallest set that has the property that it has at least 1000 more subsets than it has elements?

   (a) 9 (b) 10 (c) 50 (d) 100 (e) 1001

6. If $w < x$ and $y < z$ with $w, x, y, z \neq 0$ then which of the following must be true for every possible value of w, x, y, z?

   (i) $\dfrac{1}{w} > \dfrac{1}{x}$

   (ii) $wy < xz$

   (iii) $w + y < x + z$

   (iv) if $z < w$ then $y < x$.

   (a) all are true (b) none are true (c) only (ii) and (iii) are true

   (d) only (i), (iii) and (iv) are true (e) only (iii) and (iv) are true

7. Suppose one had six cards, each of which is colored red, yellow or blue on one side. The other side of each card has one of the symbols $\circ$, $\triangle$, or * on it. Consider this statement: "Every yellow card has a * on the other side." To prove or disprove the statement, which of the following card(s) must be turned over and checked?

1 2 3 4 5 6

   (a) card (3) only (b) cards (3) and (6) only

   (c) cards (2), (3) and (4) only (d) cards (2), (3) and (6) only

   (e) cards (2), (3), (4), and (6) only

8. If w, x, y, and z are natural numbers, $w|y$ (w divides y) and $x|z$ then which of the followings are always true?

   (i) $wx|yz$ (ii) $(w + x)|(y + z)$ (iii) $w|yz$ (iv) $wx|xy$ (v) $x^w|z^y$

   (a) all of them (b) all but (ii) (c) all but (v)

   (d) only (i), (iii) and (iv) (e) only (i)

9. A number will be said to be type T if it can be obtained as the result in step 3 of the following process:

   1. Start with a 3-digit number (ABC); for example, 378.
   2. Subtract the sum of the digits from the number. (In the example: 378-18=360.)
   3. Divide the result by 9. (In the example: 360/9= 40)

   The example shows that 40 is of type T. What is the number closest to 40 that is NOT of type T?

   (a) 32 (b) 33 (c) 38 (d) 43 (e) 44

10. Which of these numbers is the greatest?

(a) 2^{3^4} (b) 2^{4^3} (c) 3^{2^4} (d) 3^{4^2} (e)4^{3^2}

11. One hundred pennies or ninety-five pennies and a nickel are two different ways to make change for one dollar. If one had 100 pennies, 4 nickels, 2 dimes and 1 quarter, how many ways could one make change for a dollar? (Assume one does not distinguish between which pennies, nickels, etc. were used.)

(a) 7 (b) 8 (c) 15 (d) 28 (e) 30

12. Solve the system of equations

$$\begin{array}{rcrcl} |x| & + & y & = & 12 \\ x & + & |y| & = & 6. \end{array}$$

What is the product of the values of x and y in the solution (x, y) ?

(a) -18 (b) 48 (c) -20 (d) 24 (e) -27

13. On June 30, 1983 the age of a woman was equal to the sum of the digits in the year of her birth. Which one of the following is closest to her age on June 30, 2003?

(a) 39 (b) 42 (c) 45 (d) 48 (e) 51

14. What is the units digit of 3^{2003}?

(a) 1 (b) 3 (c) 7 (d) 9 (e) none of these

15. In one county 3/4 of all females are not married, and 4/5 of all males are not married. What proportion of all people are married in the county? (Assume that every marriage is between one man and one woman.)

(a) 9/40 (b) 4/9 (c) 7/20 (d) 1/3 (e) 2/9

16. How many of the 900 three digit numbers are NOT divisible by either 5 or 7?

(a) 282 (b) 566 (c) 592 (d) 618 (e) none of these

17. A rancher has 600 yards of fencing for a rectangular pen divided into four equal sections (see figure). What is the largest area he can fence for his pen?

(a) 10,000 yds^2 (b) 9,000 yds^2 (c) 6,000 yds^2 (d) 22,500 yds^2 (e) none of these

18. In the system of equations $x^7y^5 = r$ and $x^4y^3 = s$ consider x, y, r, and s to be positive. When solved for x and y in terms of r and s we get $x = r^a s^b$ and $y = r^c s^d$ for some a, b, c, and d. What is the sum $a + b + c + d$?

(a) 0 (b) 1 (c) 10 (d) -10 (e) none of these

19. Given that $x(y-3) = 2y - z$, which of the following may be FALSE?

(i) if $z = 6$ and $y \neq 3$, then $x = 2$;

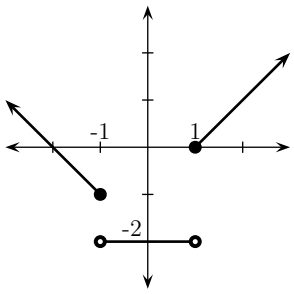
(ii) if $2y = z$, then $x = 0$;

(iii) if $x = 0$ and $y \neq 3$, then $z = 3$;

(iv) if $x \neq 2$, then $y = \dfrac{3x - z}{x - 2}$.

(a) only (ii) and (iii) (b) only (ii) and (iv) (c) only (iii) and (iv)
(d) only (ii), (iii) and (iv) (e) all of them

20. Which of the following statements are TRUE about the graph of function f (see picture)?
(i) The domain (i.e. set of possible x values) of the function is all real numbers.
(ii) If $x \leq -1$ then $f(x) \geq -1$.
(iii) If $f(x) < 0$ then $|x| < 1$.



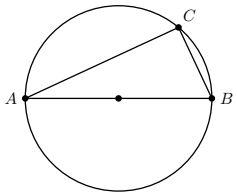
(a) all are true (b) none are true (c) only (i) and (ii) are true
(d) only (i) and (iii) are true (e) only (ii) and (iii) are true

21. Which of the numbers below is closest to the value of the sum

$$\frac{1}{\sqrt{11}+\sqrt{9}}+\frac{1}{\sqrt{13}+\sqrt{11}}+\frac{1}{\sqrt{15}+\sqrt{13}}+\ldots+\frac{1}{\sqrt{79}+\sqrt{77}}+\frac{1}{\sqrt{81}+\sqrt{79}}?$$

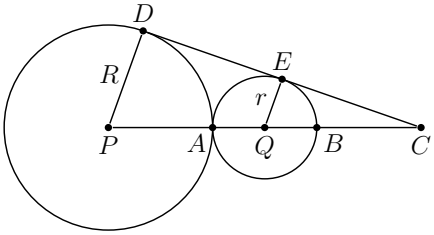
(a) 2 (b) 2.5 (c) 3 (d) 3.5 (e) 4

22. Triangle ABC is inscribed in a circle with side AB going through the center. The ratio of side AC to side BC is 2. Which of the following is closest to the ratio of the area of the circle to the area of $\triangle ABC$?



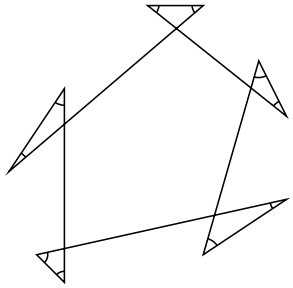
(a) $\frac{4\pi}{5}$ (b) $\frac{3\pi}{4}$ (c) π (d) $\frac{4\pi}{3}$ (e) $\frac{5\pi}{4}$

23. A circle with radius R and center P is tangent to a second circle of radius r (where $r < R$) and center Q. The common tangent line drawn on the picture intersects line PQ at point C. What is the distance of C from the smaller circle? (i.e. find the length of BC in terms of R and r.)



(a) $\frac{r^2}{R-r}$ (b) $\frac{2r^2}{R-r}$ (c) $\frac{2r^2+Rr}{r-R}$ (d) $\frac{2r^2}{r-R}$ (e) $\frac{r^2+Rr}{R-r}$

24. Which of the following is closest to the total of the marked angles on the picture?



(a) 300° (b) 350° (c) 400° (d) 450° (e)500°

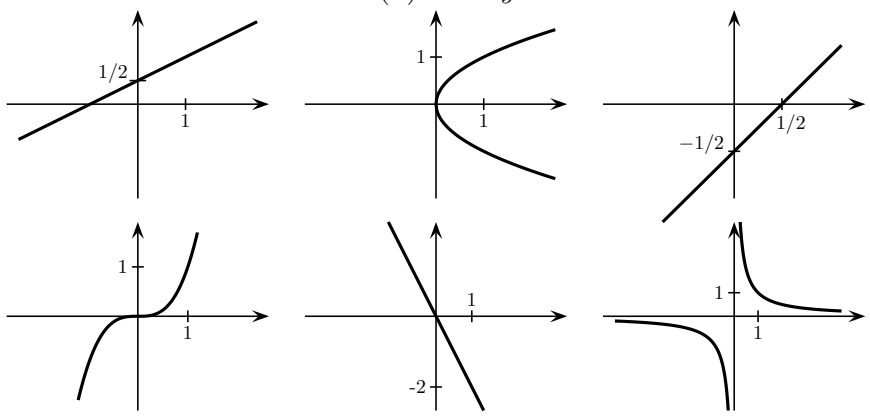
25. When $200 \times 201 \times 202 \times \ldots \times 210$ is rewritten in the form of $2^n \cdot m$, where m is odd, what is the value of n?

(a) 5 (b) 6 (c) 8 (d) 10 (e) 12

26. How many of the following six equations have a graph that matches one of the graphs shown?

(i) $y = x - \frac{1}{2}$ (ii) $y = x^3$ (iii) $y = \frac{1}{2}x + 1$

(iv) $xy = 1$ (v) $2x + y = 0$ (vi) $x = y^2$



(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

27. Line L is the perpendicular bisector of line segment PQ, where $P = (2, 3)$, $Q = (8, 11)$. What is the y-intercept of line L?

(a) $\frac{43}{4}$ (b) $\frac{41}{3}$ (c) $-\frac{1}{4}$ (d) $\frac{1}{3}$ (e) none of these

28. One January there were exactly 4 Mondays and 4 Fridays. January 1 of that year was on what day of the week?

(a) Monday (b) Tuesday (c) Wednesday (d) Thursday (e) Friday

29. A king gave all his castles to his 7 sons before he died. The youngest son received some number of castles, the second youngest received twice as many, the third three times as many, etc. so that the oldest son received seven times as many castles as the youngest. The queen, however, did not find this fair and ordered each son to give two of his castles to every other son that was younger than himself. After this all seven princes had the same number of castles. How many castles did the old king originally have?

(a) 98 (b) 126 (c) 105 (d) 84 (e) 112

30. If it takes J hours for John to do a job alone, M hours for Mike to do the same job, and T hours for the two of them working together, which of the following must be true?

(a) $\frac{T}{M+J} = 1$ (b) $\frac{M+J}{T} = 1$ (c) $\frac{T}{M} + \frac{T}{J} = 1$

(d) $T(M + J) = 1$ (e) none of these