

2004 STATE MATH CONTEST

SOLUTIONS – GRADES 7-9

1. A dividend is 6 times the divisor, and the divisor is 6 times the quotient. The dividend is equal to

(a) 6 (b) 18 (c) 36 (d) 216 (e) none of these

Solution:

Since the dividend is 6 times the divisor, the quotient must be 6. The divisor is 6 times the quotient, or 36. Thus the dividend must be $6 \times 36 = 216$. The correct answer is (d).

2. If $\frac{2a - b + c}{3} < \frac{3a + 2b - c}{2}$ then

(a) $b < \frac{5c - 5a}{8}$ (b) $b > \frac{5c - 5a}{8}$ (c) $b > \frac{5a - 5c}{4}$ (d) $b < \frac{5a - 5c}{8}$ (e) $b > \frac{5c - 5a}{4}$

Solution:

After multiplying both sides of the inequality by 6 and collecting like terms, we can obtain $-8b < 5a - 5c$. When dividing both sides of this inequality by -8 , the direction of the inequality must be reversed, which yields $b > \frac{5c - 5a}{8}$. The correct answer is (b).

3. Three kids share a basket of apples. Lisa gets half of the apples and two more, then Ann gets half of the remaining apples and two more, and finally Mary gets half of the remaining apples and two more. One apple is left over in the basket for you. How many apples were there originally in the basket?

(a) 16 (b) 20 (c) 76 (d) 36 (e) none of these

Solution:

I. One solution is to write an equation with x denoting the number of apples originally in the basket. After Lisa takes out half the apples and two more, there are $\frac{1}{2}x - 2$ apples left for Ann. Repeating these steps yields the equation $\frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2}x - 2 \right) - 2 \right) - 2 = 1$.

That is equivalent with $\frac{1}{8}x - \frac{7}{2} = 1$, from which $x = 36$ follows easily. The correct answer is (d).

II. It is easier to solve this problem with using no algebra, but working backwards. How many apples did Mary find in the basket? The one left over plus the two equals half the apples she has found, there must have been 6 before she took her share of $3+2$. Similarly, Ann must have found 16 apples in the basket, and she took $8+2$ to leave 6 for Mary. Finally, Lisa had 36 apples to begin with, she took $18+2$, and left 16 for Ann. There were originally 36 apples in the basket. The correct answer is (d).

4. For a particular medicine, the correct dosage is proportional to the weight of a patient. If a patient weighing 144 pounds should receive 96 milligram dose, how many milligrams should a 180 lb patient receive?

(a) 84.8 (b) 120 (c) 144 (d) 270 (e) none of these

Solution:

A standard example for proportional reasoning. One can solve either $\frac{144}{96} = \frac{180}{x}$ or $\frac{180}{144} = \frac{x}{96}$ or several other equations all yielding $x = 120$. The correct answer is (b).

5. If the area of of a square is numerically equal to its perimeter then what is the length of a diagonal of that square?

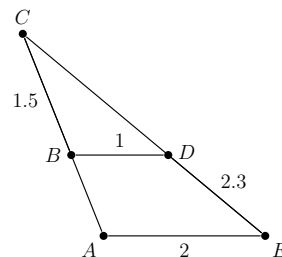
(a) $\frac{4}{\sqrt{2}}$ (b) 4 (c) $4\sqrt{2}$ (d) 16 (e) none of these

Solution:

First we need to determine the length a of a side of the square. Since the perimeter and the area are equal we must have $4 \times a = a \times a$, from which $a = 4$. The diagonal is $\sqrt{2}$ times the side, $4\sqrt{2}$. (One can also use the Pythagorean theorem to calculate this.) The correct answer is (c).

6. As pictured, segments \overline{BD} and \overline{AE} are parallel. If the lengths are as indicated, how much longer is \overline{CD} than \overline{AB} ?

(a) 0.6 (b) 0.7 (c) 0.8 (d) 0.9 (e) 1.0



Solution:

I. Triangles CBD and CAE are similar because their angles are congruent (angles CBD and CAE are corresponding angles as well as angles CDB and CEA). Therefore their corresponding sides are proportional, and since $AE = 2BD$, we must have $CA = 2CB$ and $CE = 2CD$. Thus, $AB = 1.5$ and $CD = 2.3$, the difference between these segments is 0.8. The correct answer is (c).

II. The only segment in triangle AEC parallel to AE and half of its length is the midsegment of the triangle, connecting the midpoints of \overline{CA} and \overline{CE} . Therefore, $AB = 1.5$ and $CD = 2.3$, the difference is 0.8 and the correct answer is (c).

7. Woodrow and Corrilla noticed that the sum of their ages, multiplied by their granddaughter's age, is 2004. Assume all ages are whole numbers. How old is their granddaughter?

(a) 6 (b) 8 (c) 10 (d) 12 (e) 14

Solution:

The prime factorization of 2004 is $2004 = 2 \times 2 \times 3 \times 167$. We can safely assume that the sum of the ages of the grandparents is somewhere between 13 and 333. The only divisor of 2004 in that range is 167, that must be the sum of the ages of the grandparents, therefore their granddaughter is 12 years old. The correct answer is (d).

8. A group of people chartered a bus for \$1800 sharing the cost equally. Unfortunately, there were 5 empty seats. Had they been able to convince 5 more people to go, the cost would have been \$4 less per person. How many people chartered the bus?

(a) 36 (b) 40 (c) 42 (d) 45 (e) 50

Solution:

If x people chartered the bus, their cost was $\frac{1800}{x}$, and for $x + 5$ people the cost is $\frac{1800}{x + 5}$. This leads to the equation $\frac{1800}{x} = \frac{1800}{x + 5} + 4$, which simplifies easily to the quadratic equation $x^2 + 5x - 2250 = 0$. The only positive solution of this is $x = 45$, which one can get by either factoring or using the quadratic formula. The correct answer is (d).

9. The sum of k consecutive positive integers is 1000, where $k \geq 2$. What is the smallest possible value of k ?

(a) 2 (b) 3 (c) 4 (d) 5 (e) 7

Solution:

k has to be at least 2, but 2 consecutive integers always add up to an odd number, since exactly one of them is odd. Can we have $k = 3$? No, the sum of three consecutive numbers would always be divisible by 3, since $x + (x + 1) + (x + 2) = 3x + 3$. Can $k = 4$ give a solution? The sum of four consecutive numbers $x, x + 1, x + 2$, and $x + 3$ would be $4x + 6$, which cannot be divisible by 4. But 1000 is divisible by 4, therefore $k = 4$ will not provide a solution. Trying further with $k = 5$, the numbers should average to 200, one can easily find the solution of $198 + 199 + 200 + 201 + 202 = 1000$. The correct answer is (d).

10. The sum of the solutions to the equation $|3x + 5| = 14$ is
 (a) $-10/3$ (b) -3 (c) 0 (d) 3 (e) none of these

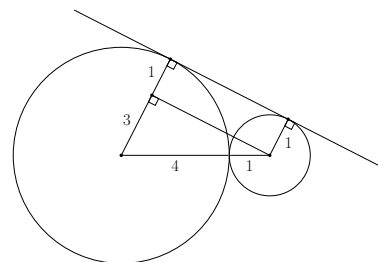
Solution:

The equation can be satisfied with either $3x + 5 = 14$ or $3x + 5 = -14$, these equations lead to the solutions of $x_1 = 3$ and $x_2 = -19/3$. The sum of the two solutions is $-10/3$, and the correct answer is (a).

11. Two circles of radii 1 and 4 units are tangent to each other from the outside. What is the length of the part of the common tangent line to both circles that is between the two different points of tangency?
 (a) 3 (b) 4 (c) 5 (d) $\sqrt{7}$ (e) none of these

Solution:

The tangent line is perpendicular to the radii at the points of tangency. Drawing a parallel line to the tangent line through the center of the smaller circle creates a right triangle. It has a hypotenuse of 5 (sum of the two radii), and a leg of 3 (difference of the two radii). Therefore the other leg must be 4 (Pythagorean theorem), and that is congruent to the segment between the points of tangency. The correct answer is (b).



12. A positive number x satisfies the equation $x = \frac{1}{1 + \frac{1}{1+x}}$. What is the value of x ?
 (a) $\frac{\sqrt{2}}{2}$ (b) $\sqrt{3} - 1$ (c) $\frac{\sqrt{5} - 1}{2}$ (d) $\frac{\sqrt{6} + 1}{4}$ (e) $\sqrt{3} - \sqrt{2}$

Solution:

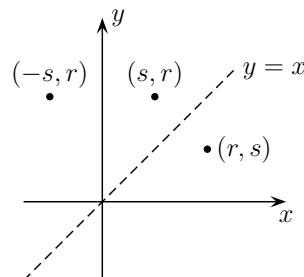
Simplifying the fraction we get $x = \frac{1}{1 + \frac{1}{1+x}} = \frac{1}{\frac{2+x}{1+x}} = \frac{1+x}{2+x}$. Multiplying both sides by $2+x$ leads to the quadratic equation $x^2 + x - 1 = 0$. This has one positive solution, $x = \frac{\sqrt{5}-1}{2}$, the correct answer is (c).

13. Point Q , which has rectangular coordinates of (r, s) , is to be reflected through the line $y = x$; then the result of that is to be reflected through the y -axis. What is the (x, y) coordinate of the final point?

(a) (s, r) (b) $(-r, s)$ (c) $(r, -s)$ (d) $(s, -r)$ (e) $(-s, r)$

Solution:

Reflecting about the $y = x$ line exchanges the coordinates, so point (r, s) changes to (s, r) . Reflecting about the y -axis multiplies the first coordinate by -1 , and leaves the second coordinate unchanged, so (s, r) goes to $(-s, r)$. The correct answer is (e).



14. Given positive numbers a, b , and c with $ab = 3$, $bc = 8$ and $ac = 6$, find abc .

(a) 8 (b) 9 (c) 12 (d) 18 (e) none of these

Solution:

I. The clever way of solving the equation system is to multiply the three equations together: $(ab)(bc)(ac) = 3 \times 8 \times 6$, from which $(abc)^2 = 144$; since all three numbers are positive, $abc = 12$. The correct answer is (c).

II. A somewhat longer method of solution is to substitute from one equation to the next. For example, solving for a and c in terms of b from the first two equations yields $a = 3/b$ and $c = 8/b$. Substituting these in the third we get $\frac{3}{b} \times \frac{8}{b} = 6$. This has one positive solution, $b = 2$, and then $a = 3/2$ and $c = 4$. The product of the numbers is $abc = 3/2 \times 2 \times 4 = 12$.

15. The word WEBER hides a 5-digit number. Different letters indicate different digits, and same letters stand for the same digits. Every digit is a prime number and so is the sum of the 5 digits. The 2-digit number EW and the 3-digit number EBR are also primes. What digit does letter B represent?

(a) 1 (b) 2 (c) 3 (d) 5 (e) 7

Solution:

The four different letters must stand for the four one-digit primes 2,3,5,7 in some order. Since the sum of these four numbers is 17, the only way the sum of the five numbers can be odd (necessary to make it prime) is if the duplicated number is 2. Therefore E stands for 2. Now W cannot be 5 or 7 (EW would not be prime), so W must be 3. Furthermore, R cannot be 5 (EBR would not be prime), so R=7, leaving B=5. Checking shows the numbers 3+2+5+2+7=19, 23 and 257 all to be primes, so B represents 5 and the correct answer is (d).

16. A can having a shape of a right circular cylinder of height 18 inches is filled with water. That water is emptied into a partially filled barrel of water having a shape of a right circular cylinder with a diameter of one yard. If the level of the water in the barrel increases by 2 inches, what is the radius of the can?

(a) $1/12$ yd (b) $1/10$ yd (c) $1/9$ yd (d) $1/8$ yd (e) $1/6$ yd

Solution:

Let the radius of the can be r inches. The volume of the can is then $18\pi r^2$ in³. This equals the volume of the extra water in the barrel after emptying the can into it. We must be careful to calculate in in³ again. The diameter of the barrel is 1 yd=36 in; therefore, its radius is 18 inches, and the volume of the added water is $2\pi 18^2$ in³. Solving $18\pi r^2 = 2\pi 18^2$ is very easy; we get only one positive solution of $r = 6$ in = $1/6$ yd. The correct answer is (e).

17. If the sum of two positive numbers is divided by the sum of their reciprocals the result will be:

(a) the product of the two numbers
(b) the reciprocal of the product of the two numbers
(c) the square of the sum divided by the product
(d) the product divided by the square of the sum
(e) the sum of the squares of the numbers.

Solution:

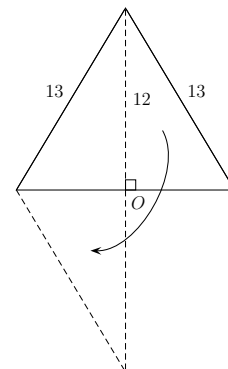
Let the two numbers be a and b . Simple algebraic manipulation yields: $\frac{a+b}{\frac{1}{a} + \frac{1}{b}} = \frac{a+b}{\frac{a+b}{ab}} = ab$. Thus, the quantity in question is the product of the two numbers; the correct answer is (a).

18. An isosceles triangle has a base of length 10 and equal sides of length 13. What other base can an isosceles triangle with equal sides of length 13 have and still have the same area as the original?

(a) 12 (b) 18 (c) 24 (d) 30 (e) no other base can give the same area

Solution:

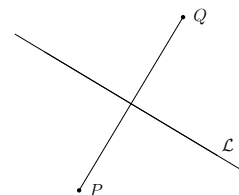
I. A geometric solution is to rotate one half of the isosceles triangle by 180° about the midpoint of the base. Clearly we get another isosceles triangle with equal sides of 13. The new base is now twice the original altitude corresponding to the base. That altitude was 12 (Pythagorean theorem), so the new base has length 24. The correct answer is (c).



II. A more algebraic solution is to calculate the area of the triangle. Just like before, the height is 12, so the area is therefore $A = \frac{10 \times 12}{2} = 60$. Now, if there is another isosceles triangle with area 60, and height x , its base will be $120/x$. The equal sides can now be calculated from the Pythagorean theorem: $x^2 + \left(\frac{120}{2x}\right)^2 = 13^2$. Simplifying this we get $x^4 - 169x^2 + 3600 = 0$. This can take some calculation to solve, unless we remember that one solution of this equation must be 12, this helps with factoring. From $(x^2 - 144)(x^2 - 25) = 0$ we get another positive solution of $x = 5$, the corresponding base of the triangle is 24.

19. Line \mathcal{L} is the perpendicular bisector of the line segment \overline{PQ} , where $P = (2, 3)$ and $Q = (8, 11)$. Find the y -intercept of the line \mathcal{L} .

(a) $43/4$ (b) 7 (c) $1/3$ (d) $41/3$ (e) none of these



Solution:

The perpendicular bisector must go through the midpoint of \overline{PQ} , which has coordinates $\left(\frac{2+8}{2}, \frac{3+11}{2}\right) = (5, 7)$. Since the slope of segment \overline{PQ} is $\frac{11-3}{8-2} = \frac{4}{3}$, and since the slope of the perpendicular bisector must be the negative reciprocal of this, the slope of \mathcal{L} is $-\frac{3}{4}$. Thus the equation of \mathcal{L} is $y = -\frac{3}{4}(x - 5) + 7 = -\frac{3}{4}x + \frac{43}{4}$. The y -intercept of the line is $\frac{43}{4}$, and the correct answer is (a).

20. Kate showed 8 closed boxes to her brother Matthew. “Look” she said, “the boxes have 7, 10, 13, 18, 28, 31, 46, and 62 marbles respectively. Some marbles are red, some are blue. It is possible to take one box away so that exactly twice as many red marbles are left as blue marbles. Can you do it?” How many marbles are in the box Matthew should take away?
- (a) 7 (b) 10 (c) 18 (d) 46 (e) 62

Solution:

The key is to notice that after taking the box away, the number of marbles left must be divisible by 3 (there are twice as many red as blue marbles). The sum of all marbles is 215, which gives a remainder of 2 when divided by 3. Therefore, the number of marbles in the box that has to be taken away must also give a remainder of 2 when divided by 3. There is only one such box, the one with 62 marbles in it. The correct answer is (e).

21. All the three digit positive integers (100 through 999) are written on identical pieces of paper and thrown into hat. If one number is selected at random, what is the probability that the sum of the digits is 4?
- (a) 9/900 (b) 10/899 (c) 1/899 (d) 8/900 (e) none of these

Solution:

We have to count how many of the 900 numbers in the hat have the sum of the digits equal to 4. Perhaps the easiest is to list them all: 103, 130, 112, 121, 202, 220, 211, 301, 310, 400. Since there are 10 such numbers, the probability of drawing one of them is 10/900. Since this is not listed, the correct answer is “none of these”, (e).

22. Flying directly against the wind a small airplane requires 16 hours to go 1440 miles. Returning with the same wind directly behind it, the plane covers the distance in 10 hours. In miles per hour, what is the speed of the plane in still air?
- (a) 110.77 (b) 117 (c) 132 (d) 144 (e) none of these

Solution:

I. Covering 1440 miles in 16 hours means a speed of 90 miles per hour. Similarly on the return trip, the speed is 144 miles per hour. The speed of the airplane is the average of these two speeds, $(90 + 144)/2 = 117$ miles per hour. The correct answer is (b).

II. Essentially the same solution can be put in a more algebraic form. Let s denote the speed of the plane with no wind, and w the speed of the wind. We have to solve the equation system

$$16(s - w) = 1440$$

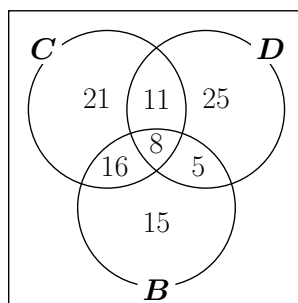
$$10(s + w) = 1440$$

The solution of this system is $s = 117$ and $w = 27$; so the speed of the plane is 117 mph.

23. A group of 123 people were surveyed with regard to owing dogs, cats, or birds. Of the group, 49 had at least one dog, 56 had at least one cat, and 44 had one or more birds. Of these who had any of the three, 19 had at least one dog and one cat, 24 had at least one cat and one bird, 13 had at least one dog and one bird and 8 had at least one of each. How many had none of the three types?
- (a) 22 (b) 26 (c) 27 (d) 30 (e) none of these

Solution:

The easiest way to organize this information is in a Venn diagram. We can represent the owners of dogs, cats and birds by three sets, and start with the intersection of all three having 8 elements. Next we can find the intersections of two sets, for example out of 19 who have cats and dogs, 8 are in the middle so 11 own only cats and dogs. Filling out the diagram we get:



The total of the people in these sets is 101; therefore, 22 people are not represented. They are the ones who have none of these three pets. The correct answer is (a).

24. The natives on the island of Ooz form two tribes: the Veracities, who always tell the truth, and the Atrocities, who never tell the truth. Each has his name tattooed on his forehead, so names are not a problem and they are all well acquainted so that they know the true state of affairs. A traveller came upon a group of four natives and asked to which tribe each belonged. He received these answers:

Au: At least one of us is an Atrocity

Bo: Au is a Veracity

Co: We are not all Atrocities

Di: All of them are lying

To which tribe does each belong?

- (a) All are Veracities (b) All are Atrocities (c) only Di is a Veracity
 (d) All but Di are Veracities (e) Au and Bo are Veracities and the other two are Atrocities

Solution:

Au must be saying the truth, if he didn't, he would be an Atrocity making his own statement true. Thus Au is Veracity, and therefore Bo's and Co's sentences are also true, making them Veracities as well. Since the first three are telling the truth, Di is lying, making him the only Atrocity in the group. The correct answer is (d).

25. The notation $a\mathcal{M}b$ is defined to mean that a is a multiple of b and that a and b are both positive integers. Which of the following statements are always true for all positive integers w, x, y and z ?

- (i) $x\mathcal{M}y$ and $x\mathcal{M}z$ implies $x\mathcal{M}(yz)$
 - (ii) $x\mathcal{M}y$ and $y\mathcal{M}z$ implies $(x+y)\mathcal{M}z$
 - (iii) $x\mathcal{M}y$ and $y\mathcal{M}z$ implies $x\mathcal{M}z$
 - (iv) $w\mathcal{M}x$ and $y\mathcal{M}z$ implies $(wy)\mathcal{M}(xz)$
- (a) all of them (b) only (i), (ii), and (iii)
 (c) only (ii), (iii), and (iv) (d) only (iii), and (iv) (e) none of these

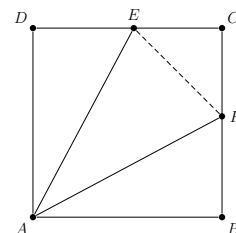
Solution:

Let's look at every statement.

- (i) Not true in general; for example, $12\mathcal{M}6$ and $12\mathcal{M}4$, but $12\mathcal{M}24$ is not true.
- (ii) This statement is true. If $x = ky$ and $y = lz$ for some integers k and l , then $x + y = ky + y = (k + 1)y = (k + 1)lz$ and thus $x + y$ is a multiple of z .
- (iii) True again. Just like before, if $x = ky$ and $y = lz$ for some integers k and l , then $x = (kl)z$, a multiple of z .
- (iv) The statement is true. If $w = kx$ and $y = lz$, then $wy = (kl)xz$.

The true statements are (ii), (iii), and (iv); answer (c) is the correct one.

26. In a square $ABCD$, points E and F are placed on the boundary so that the line segments \overline{AE} and \overline{AF} divide the square into three parts of equal area. What is the ratio of the area of the triangle AEF to that of the square?
- (a) $3/18$ (b) $4/18$ (c) $5/18$ (d) $6/18$ (e) $7/18$



Solution:

For $\triangle ABF$ to have $1/3$ of the area of the square, BF must be $2/3$ of BC . Similarly, $DE = (2/3)DC$. Hence, the area of $\triangle ECF$ is $1/18$ of the area of the square. Since quadrilateral $AFCE$ covers $1/3$ of the square, and $1/18$ of the square is to be removed, $\triangle AEF$ covers $1/3 - 1/18 = 5/18$ of the square. The correct answer is (c).

27. Tommy's house is next to a road that goes straight to Tommy's school. A railway runs parallel to that road. When Tommy leaves home at his regular time the moving train just catches him as he reaches school. One day Tommy left 4 minutes 10 seconds later and the train caught him one mile before he reached school. Given that Tommy always bikes at 12 miles per hour, the train always leaves on time, and the train always travels at the same constant speed, which of the following is the closest to the speed of the train?
- (a) 50 mph (b) 55 mph (c) 60 mph (d) 65 mph (e) 70 mph

Solution:

On the day when Tommy was late, and the train caught him earlier, he had 1 mile to bike to school from the meeting place, that takes him 5 minutes (given his speed of 12 mph). Since he left 4 minutes 10 seconds late, the moment of meeting the train was 50 seconds earlier than the usual time he (and the train) arrives at the school. Therefore, it takes the train 50 seconds or $50/3600$ hours to travel 1 mile, so its speed is $1 \div (50/3600) = 72$ mph. The correct answer is (e).

28. Write down all the integers from 1 to 30 to form the number

1234567891011121314...2930.

Now, delete 44 digits from this number and call the resulting number N . What is the possible value of N that is closest (smaller, larger or equal) to $5 \cdot 10^6$?

- (a) 5,001,220 (b) 4,998,930 (c) 4,999,888 (d) 5,000,111 (e) none of these

Solution:

We have started with a 51 digit number. After deleting 44 of those digits we will have a 7 digit number left. We need to figure out which 44 digits to delete to get close to 5,000,000. It is easier to think of how to select 7 of the digits (without changing their order) to get close to 5,000,000. We do not have enough 0's to get exactly 5,000,000. If we try to get close to the number from below, we have to use a 4 as the first digit. Then we can use two 9's (one from 9, one from 19), but the last 9 from 29 cannot be used at the thousands place, as there are not enough digits left after it. The largest number we can have less than 5,000,000 is therefore 4,998,930. On the other hand if we try to get close from above, we have to start with 5,0,0 but now a 1 must be used, and then 2's before the last digit of the sequence can be added. The smallest number greater than 5,000,000 is therefore 5,001,220. The closest to 5,000,000 is 4,998,930 the correct answer is (b).

29. The center of a cube is reflected about every face of the cube, and the resulting points are connected to the nearest vertices of the cube. How many edges does the new solid have?

(a) 24 (b) 30 (c) 32 (d) 36 (e) 48

Solution:

At the first glance we are placing a square based pyramid on every face of the cube, adding $6 \times 4 = 24$ additional edges to the existing 12. However, the original 12 edges of the cube will disappear. They are at the intersection of two triangular faces of adjacent pyramids, and those faces happen to be in the same plane, because of the reflections about the perpendicular faces. The solid has only 24 edges, the correct answer is (a).

30. Five friends Alex, Bill, Charlie, Daniel, and Eddy all own different brands of cars. Their cars are a Ford, a Buick, a Honda, an Audi and a Mercedes. One week they decided to try each others' cars and they traded cars every day to try all 5 cars. From Monday through Thursday none drove his own car and on Friday everybody drove his own car.

On Monday Daniel drove the Audi.

On Tuesday Charlie drove the Mercedes.

On Wednesday Charlie drove the Honda and Eddy the Mercedes.

On Thursday Alex had the Audi and Daniel had the Buick.

What kind of car does Bill have?

(a) Ford (b) Buick (c) Honda (d) Audi (e) Mercedes

Solution:

Let's start with Wednesday. The Honda and the Mercedes are used by Charlie and Eddy, Daniel must have driven one of the other three cars. He drove the Audi and Buick on other days, so Daniel must have driven the Ford on Wednesday. Since Alex drove the Audi on Thursday, that leaves Alex the Buick and Bill the Audi on Wednesday.

On Thursday, the Audi and the Buick are occupied, Charlie drove the Mercedes and the Honda on other days, he must have driven the Ford on Thursday. Who drove the Mercedes? Eddy didn't (he drove it on Wednesday), so Bill must have driven the Mercedes, and Eddy is left with the Honda.

What did Charlie drive on Monday? Not the Audi (Daniel drove it), and none of the Mercedes, Honda or Ford, since he drove them on other days. He must have driven the Buick on Monday. Similarly, Eddy must have driven the Ford, Bill the Honda and Alex the Mercedes.

Tuesday is now easy to figure out, Daniel drove the Honda, Alex the Ford, Bill the Buick and Eddy the Audi. Since Bill drove every car but the Ford during the first four days, he must have a Ford, the correct answer is (a).

One can organize the information into a 5x5 table and fill it out in one minute, following the above or some similar logic.