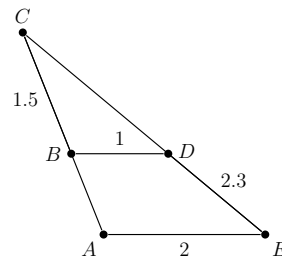


2004 STATE MATH CONTEST

GRADES 7 – 9

1. A dividend is 6 times the divisor, and the divisor is 6 times the quotient. The dividend is equal to
 (a) 6 (b) 18 (c) 36 (d) 216 (e) none of these
2. If $\frac{2a - b + c}{3} < \frac{3a + 2b - c}{2}$ then
 (a) $b < \frac{5c - 5a}{8}$ (b) $b > \frac{5c - 5a}{8}$ (c) $b > \frac{5a - 5c}{4}$ (d) $b < \frac{5a - 5c}{8}$ (e) $b > \frac{5c - 5a}{4}$
3. Three kids share a basket of apples. Lisa gets half of the apples and two more, then Ann gets half of the remaining apples and two more, and finally Mary gets half of the remaining apples and two more. One apple is left over in the basket for you. How many apples were there originally in the basket?
 (a) 16 (b) 20 (c) 76 (d) 36 (e) none of these
4. For a particular medicine, the correct dosage is proportional to the weight of a patient. If a patient weighing 144 pounds should receive 96 milligram dose, how many milligrams should a 180 lb patient receive?
 (a) 84.8 (b) 120 (c) 144 (d) 270 (e) none of these
5. If the area of a square is numerically equal to its perimeter then what is the length of a diagonal of that square?
 (a) $\frac{4}{\sqrt{2}}$ (b) 4 (c) $4\sqrt{2}$ (d) 16 (e) none of these

6. As pictured, segments \overline{BD} and \overline{AE} are parallel. If the lengths are as indicated, how much longer is \overline{CD} than \overline{AB} ?
 (a) 0.6 (b) 0.7 (c) 0.8 (d) 0.9 (e) 1.0



7. Woodrow and Corrilla noticed that the sum of their ages, multiplied by their granddaughter's age, is 2004. Assume all ages are whole numbers. How old is their granddaughter?

(a) 6 (b) 8 (c) 10 (d) 12 (e) 14

8. A group of people chartered a bus for \$1800 sharing the cost equally. Unfortunately, there were 5 empty seats. Had they been able to convince 5 more people to go, the cost would have been \$4 less per person. How many people chartered the bus?

(a) 36 (b) 40 (c) 42 (d) 45 (e) 50

9. The sum of k consecutive positive integers is 1000, where $k \geq 2$. What is the smallest possible value of k ?

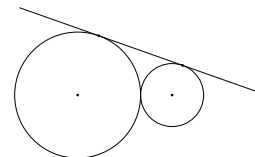
(a) 2 (b) 3 (c) 4 (d) 5 (e) 7

10. The sum of the solutions to the equation $|3x + 5| = 14$ is

(a) $-10/3$ (b) -3 (c) 0 (d) 3 (e) none of these

11. Two circles of radii 1 and 4 units are tangent to each other from the outside. What is the length of the part of the common tangent line to both circles that is between the two different points of tangency?

(a) 3 (b) 4 (c) 5 (d) $\sqrt{7}$ (e) none of these



12. A positive number x satisfies the equation $x = \frac{1}{1 + \frac{1}{1 + x}}$. What is the value of x ?

(a) $\frac{\sqrt{2}}{2}$ (b) $\sqrt{3} - 1$ (c) $\frac{\sqrt{5} - 1}{2}$ (d) $\frac{\sqrt{6} + 1}{4}$ (e) $\sqrt{3} - \sqrt{2}$

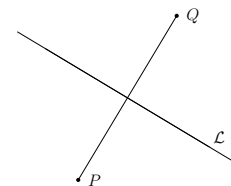
13. Point Q , which has rectangular coordinates of (r, s) , is to be reflected through the line $y = x$; then the result of that is to be reflected through the y -axis. What is the (x, y) coordinate of the final point?

(a) (s, r) (b) $(-r, s)$ (c) $(r, -s)$ (d) $(s, -r)$ (e) $(-s, r)$

14. Given positive numbers a, b , and c with $ab = 3$, $bc = 8$ and $ac = 6$, find abc .

(a) 8 (b) 9 (c) 12 (d) 18 (e) none of these

15. The word WEBER hides a 5-digit number. Different letters indicate different digits, and same letters stand for the same digits. Every digit is a prime number and so is the sum of the 5 digits. The 2-digit number EW and the 3-digit number EBR are also primes. What digit does letter B represent?
- (a) 1 (b) 2 (c) 3 (d) 5 (e) 7
16. A can having a shape of a right circular cylinder of height 18 inches is filled with water. That water is emptied into a partially filled barrel of water having a shape of a right circular cylinder with a diameter of one yard. If the level of the water in the barrel increases by 2 inches, what is the radius of the can?
- (a) $1/12$ yd (b) $1/10$ yd (c) $1/9$ yd (d) $1/8$ yd (e) $1/6$ yd
17. If the sum of two positive numbers is divided by the sum of their reciprocals the result will be:
- (a) the product of the two numbers
 (b) the reciprocal of the product of the two numbers
 (c) the square of the sum divided by the product
 (d) the product divided by the square of the sum
 (e) the sum of the squares of the numbers.
18. An isosceles triangle has a base of length 10 and equal sides of length 13. What other base can an isosceles triangle with equal sides of length 13 have and still have the same area as the original?
- (a) 12 (b) 18 (c) 24 (d) 30 (e) no other base can give the same area
19. Line \mathcal{L} is the perpendicular bisector of the line segment \overline{PQ} , where $P = (2, 3)$ and $Q = (8, 11)$. Find the y -intercept of the line \mathcal{L} .
- (a) $43/4$ (b) 7 (c) $1/3$ (d) $41/3$ (e) none of these
20. Kate showed 8 closed boxes to her brother Matthew. "Look" she said, "the boxes have 7, 10, 13, 18, 28, 31, 46, and 62 marbles respectively. Some marbles are red, some are blue. It is possible to take one box away so that exactly twice as many red marbles are left as blue marbles. Can you do it?" How many marbles are in the box Matthew should take away?
- (a) 7 (b) 10 (c) 18 (d) 46 (e) 62
21. All the three digit positive integers (100 through 999) are written on identical pieces of paper and thrown into hat. If one number is selected at random, what is the probability that the sum of the digits is 4?
- (a) $9/900$ (b) $10/899$ (c) $1/899$ (d) $8/900$ (e) none of these



22. Flying directly against the wind a small airplane requires 16 hours to go 1440 miles. Returning with the same wind directly behind it, the plane covers the distance in 10 hours. In miles per hour, what is the speed of the plane in still air?

(a) 110.77 (b) 117 (c) 132 (d) 144 (e) none of these

23. A group of 123 people were surveyed with regard to owning dogs, cats, or birds. Of the group, 49 had at least one dog, 56 had at least one cat, and 44 had one or more birds. Of these who had any of the three, 19 had at least one dog and one cat, 24 had at least one cat and one bird, 13 had at least one dog and one bird and 8 had at least one of each. How many had none of the three types?

(a) 22 (b) 26 (c) 27 (d) 30 (e) none of these

24. The natives on the island of Ooz form two tribes: the Veracities, who always tell the truth, and the Atrocities, who never tell the truth. Each has his name tattooed on his forehead, so names are not a problem and they are all well acquainted so that they know the true state of affairs. A traveller came upon a group of four natives and asked to which tribe each belonged. He received these answers:

Au: At least one of us is an Atrocity

Bo: Au is a Veracity

Co: We are not all Atrocities

Di: All of them are lying

To which tribe does each belong?

- (a) All are Veracities (b) All are Atrocities (c) only Di is a Veracity
(d) All but Di are Veracities (e) Au and Bo are Veracities and the other two are Atrocities

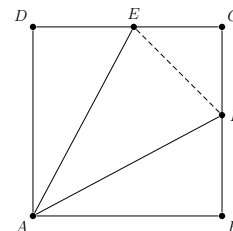
25. The notation $a\mathcal{M}b$ is defined to mean that a is a multiple of b and that a and b are both positive integers. Which of the following statements are always true for all positive integers w, x, y and z ?

- (i) $x\mathcal{M}y$ and $x\mathcal{M}z$ implies $x\mathcal{M}(yz)$
(ii) $x\mathcal{M}y$ and $y\mathcal{M}z$ implies $(x+y)\mathcal{M}z$
(iii) $x\mathcal{M}y$ and $y\mathcal{M}z$ implies $x\mathcal{M}z$
(iv) $w\mathcal{M}x$ and $y\mathcal{M}z$ implies $(wy)\mathcal{M}(xz)$

- (a) all of them (b) only (i), (ii), and (iii)
(c) only (ii), (iii), and (iv) (d) only (iii), and (iv) (e) none of these

26. In a square $ABCD$, points E and F are placed on the boundary so that the line segments \overline{AE} and \overline{AF} divide the square into three parts of equal area. What is the ratio of the area of the triangle AEF to that of the square?

(a) $3/18$ (b) $4/18$ (c) $5/18$ (d) $6/18$ (e) $7/18$



27. Tommy's house is next to a road that goes straight to Tommy's school. A railway runs parallel to that road. When Tommy leaves home at his regular time the moving train just catches him as he reaches school. One day Tommy left 4 minutes 10 seconds later and the train caught him one mile before he reached school. Given that Tommy always bikes at 12 miles per hour, the train always leaves on time, and the train always travels at the same constant speed, which of the following is the closest to the speed of the train?

(a) 50mph (b) 55mph (c) 60mph (d) 65mph (e) 70mph

28. Write down all the integers from 1 to 30 to form the number

1234567891011121314...2930.

Now, delete 44 digits from this number and call the resulting number N . What is the possible value of N that is closest (smaller, larger or equal) to $5 \cdot 10^6$?

(a) 5,001,220 (b) 4,998,930 (c) 4,999,888 (d) 5,000,111 (e) none of these

29. The center of a cube is reflected about every face of the cube, and the resulting points are connected to the nearest vertices of the cube. How many edges does the new solid have?

(a) 24 (b) 30 (c) 32 (d) 36 (e) 48

30. Five friends Alex, Bill, Charlie, Daniel, and Eddy all own different brands of cars. Their cars are a Ford, a Buick, a Honda, an Audi and a Mercedes. One week they decided to try each others' cars and they traded cars every day to try all 5 cars. From Monday through Thursday none drove his own car and on Friday everybody drove his own car.

On Monday Daniel drove the Audi.

On Tuesday Charlie drove the Mercedes.

On Wednesday Charlie drove the Honda and Eddy the Mercedes.

On Thursday Alex had the Audi and Daniel had the Buick.

What kind of car does Bill have?

(a) Ford (b) Buick (c) Honda (d) Audi (e) Mercedes