

# 2004 STATE MATH CONTEST

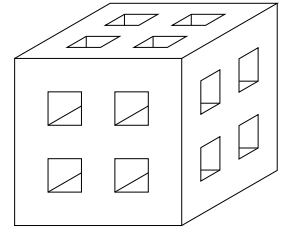
## JUNIOR PRETEST

1. Find the value of the alternating sum  $1 - 2 + 3 - 4 + \dots + 2003 - 2004$ .

(a) -501                      (b) -1002                      (c) -2003                      (d) - 2505                      (e) 1002

2. In a 5 by 5 by 5 cube, four symmetrically placed 1 by 1 square holes are cut perpendicular to each face all the way through to the opposite face. (See the picture at the right.) How many small 1 by 1 by 1 cubes are left after this construction?

(a) 81                      (b) 61                      (c) 75                      (d) 105                      (e) 96



3. If a certain number is both subtracted from the numerator and added to the denominator of the fraction  $\frac{29}{31}$ , the resulting fraction simplifies to  $\frac{1}{7}$ . The number is closest to:

(a) 20.3                      (b) 21.3                      (c) 22.3                      (d) 23.3                      (e) 24.3

4. Five years ago a girl was one fifth the age of her mother. In twenty two years she will be half as old as her mother. Find the sum of the ages of the girl and her mother.

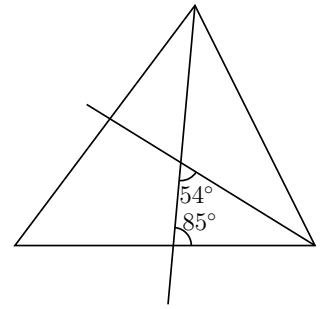
(a) 42 yrs                      (b) 46 yrs                      (c) 50 yrs                      (d) 54 yrs                      (e) none of these

5. A sphere is circumscribed around a rectangular box having dimensions 4 by 12 by 18. What is the radius of this sphere?

(a) 10                      (b) 11                      (c) 12                      (d) 13                      (e) 14

6. An angle bisector of a triangle forms an angle of  $85^\circ$  with the opposite side and an angle of  $54^\circ$  with another angle bisector, as pictured. How large is the smallest angle of the triangle?

(a)  $20^\circ$       (b)  $26^\circ$       (c)  $32^\circ$       (d)  $34^\circ$       (e)  $40^\circ$



7. One barrel is full of oil, and an identical barrel is half full. The full barrel weighs 86kg, and the half full barrel weighs 53kg. What is the weight of an empty barrel?

(a) 12      (b) 14      (c) 16      (d) 18      (e) 20

8. Four children guess the height of their teacher to be 196cm, 163cm, 178cm, and 185cm. They all missed the exact height and they were off (in some order) by 1 cm, 6cm, 16cm and 17cm. What is the sum of the digits in the teacher's real height?

(a) 15      (b) 16      (c) 17      (d) 18      (e) 19

9. Suppose that all three digit numbers (100 through 999) were written individually on pieces of paper and thrown in a hat. At least how many pieces of paper have to be randomly selected, so that without looking at these, one can be certain that at least two of the pieces of paper selected will have numbers whose digits sum to the same thing?

(a) 10      (b) 18      (c) 28      (d) 40      (e) 50

10. The symbol  $+$  is for addition and  $\times$  is for multiplication. If one were to leave the “ $+$ ” and “ $\times$ ” signs where they are in  $2 \times 3 + 5$  (which total 11) but interchange digits and/or insert grouping symbols as in  $2 \times (3 + 5)$  (which totals 16) and  $3 \times 5 + 2$  (which totals 17), how many different values would be possible?

(a) 4      (b) 6      (c) 8      (d) 10      (e) none of these

11. The six digit number  $15a64b$  is divisible by 45. Which one of the following could (not must) be the product of the two missing digits  $a$  and  $b$ ?

(a) 12                      (b) 30                      (c) 2                      (d) 20                      (e) 10

12. The gas mileage of a car is

16 miles per gallon at 80 miles per hour,

18 miles per gallon at 70 miles per hour,

20 miles per gallon at 60 miles per hour.

If that car were driven at 80 mph for 2 hours, at 70 mph for 3 hours, and at 60 mph for 5 hours, about how many gallons of gas would be used?

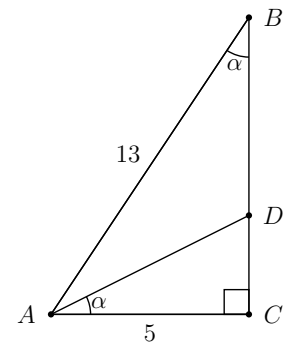
(a) 35                      (b) 37                      (c) 39                      (d) 41                      (e) 43

13. The graph of  $y = ax^2 + bx + 4$  passes through  $(x, y) = (0, 4)$  for all values of  $a$  and  $b$ . Determine  $a$  and  $b$  such that the graph also passes through  $(x, y) = (1, 3)$  and  $(x, y) = (2, 6)$ . The value of  $a + b$  is:

(a) 1                      (b) 2                      (c) 0                      (d) -2                      (e) -1

14. In  $\triangle ACB$  at the right,  $\angle C = 90^\circ$ ,  $\overline{AC} = 5$  and  $\overline{AB} = 13$ . A line is drawn from  $A$  to  $D$  on  $\overline{BC}$  making an angle of measure  $\alpha$  with  $\overline{AC}$ , which just happens to be the measure of  $\angle B$ . Compute the ratio of  $\overline{BD}$  to  $\overline{CD}$ .

(a)  $\frac{119}{25}$                       (b) 4                      (c)  $\frac{144}{25}$                       (d) 5                      (e)  $\frac{169}{25}$



15. Which of the following are always true for all nonzero real numbers  $x, y$  and  $z$ .

- i.  $(x \cdot y) \div z = x \cdot (y \div z)$
- ii.  $(x \div y) \div z = x \div (y \div z)$
- iii.  $(x \cdot y) \cdot z = x \cdot (y \cdot z)$
- iv.  $(x \div y) \cdot z = x \div (y \div z)$
- v.  $x \div (y \cdot z) = x \div (y \div z)$

(a) only i, iii      (b) only i, iv      (c) only ii, iii, iv      (d) only i, iii, iv      (e) all of the above