

# 2005 STATE MATH CONTEST

## GRADES 7 – 9

1. Jimmy decided to tell the truth on Mondays, Wednesdays and Fridays, but lie every other day. One day he says: "I will tell the truth tomorrow." What day of the week did he make this statement?  
  
(a) Monday      (b) Thursday      (c) Saturday      (d) Sunday      (e) none of these
2. The planes containing the faces of a cube divide the space into several regions. How many regions?  
  
(a) 6                      (b) 12                      (c) 24                      (d) 27                      (e) 36
3. In a recent election 65% of the eligible voters voted. The 18–21 year olds made up 26% of the eligible voters and 25% of them voted. What percentage of those who actually voted were 18–21 year olds?  
  
(a) 8%                      (b) 10%                      (c) 12%                      (d) 14%                      (e) none of these
4. Which one of the following is a factor of  $30x^2 + 11xy - 30y^2$ ?  
  
(a)  $6x + 5y$       (b)  $5x - 6y$       (c)  $5x + 3y$       (d)  $3x - 5y$       (e) none of these
5. The function  $f(x) = -x^2 + px + q$  has a maximum of 1 and a root (zero) of 2. What is the sum of the values of  $p$  that fit those conditions?  
  
(a) -4                      (b) 1                      (c) 5                      (d) 8                      (e) 10
6. Sam is moving to a colder climate. Right now his car's 20 quart radiator contains a mixture of 20% antifreeze and 80% water. At his new location he will need the mixture to be 50% antifreeze. How many quarts of his current mixture does he need to change for pure antifreeze to reach the required 50% concentration?  
  
(a) 7.5                      (b) 2.5                      (c) 5                      (d) 10                      (e) none of these

7. If we add two angles along every side of a pentagon (going around in order), we get  $198^\circ$ ,  $210^\circ$ ,  $202^\circ$ ,  $220^\circ$ , and  $250^\circ$ . What is the smallest angle of the pentagon?
- (a)  $74^\circ$                       (b)  $80^\circ$                       (c)  $88^\circ$                       (d)  $95^\circ$                       (e)  $110^\circ$
8. A triangle and a quadrilateral intersect in finitely many points. What is the most number of intersection points they can have?
- (a) 3                      (b) 4                      (c) 6                      (d) 8                      (e) none of these
9. Miss Black, Mr. Crimson, Mrs. Gold, Mr. Green, and Mr. White each own a car that has a color that is the name of one of the other four. Mr. Green's sister is married to the owner of the crimson car. The husband of the owner of the white car carpool with the owner of the green car, who in turn is engaged to Miss Black. Who owns the black car?
- (a) Miss Black    (b) Mr. Crimson    (c) Mrs. Gold    (d) Mr. Green    (e) Mr. White
10. What is the smallest value of  $n$  such that the factorial of  $n$  is divisible by 414?
- (a) 5                      (b) 401                      (c) 1003                      (d) 2005                      (e) none of these
11. Sets  $A$ ,  $B$ ,  $C$  and  $D$  are all subsets of quadrilaterals.  $A$  is the set of rhombi,  $B$  is the set of rectangles,  $C$  is the set of parallelograms, and  $D$  is the set of kites. What is the set  $(A \cap B) \cup (C \cap D)$ ?
- (a) squares    (b) rectangles    (c) parallelograms    (d) rhombi    (e) none of these
12. We wrote the numbers  $1, 2, 3, \dots, 20$  on the blackboard. In one step we erase two of the numbers  $a$  and  $b$  and write  $a + b - 1$  on the board instead. After repeating this step 19 times, there will be only one number on the board. What is that number?
- (a) 190                      (b) 191                      (c) 209                      (d) 210                      (e) none of these
13. Suppose you could trade your current job for a new job that pays 40% more per hour, but would require 30% fewer hours of work per week. How would your new weekly salary compare to your current one?
- (a) 10% more    (b) 10% less    (c) 2% more    (d) 2% less    (e) none of these

14. What is the sum of the solutions of the equation  $|2 - |1 - x|| = 1$ ?

- (a) -2                      (b) 0                      (c) 4                      (d) 6                      (e) none of these

15. The graphs of  $y = 2^{4x^2+2x+4}$  and  $y = 4^{8x^2+2x+1}$  intersect in two points:  $(x_1, y_1)$  and  $(x_2, y_2)$ . The value of  $x_1x_2$  is:

- (a) -1/2                      (b) 1/3                      (c) -1/6                      (d) 1/8                      (e) none of these

16. What is the value of  $\frac{a+b}{a-b} - \frac{a-b}{a+b}$ , if  $a = x + y$ ,  $b = x - y$  and  $x, y \neq 0$ ?

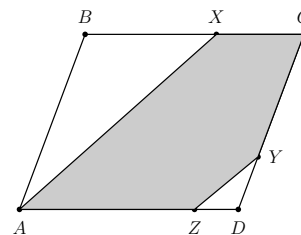
- (a)  $\frac{x^2 - y^2}{xy}$                       (b)  $\frac{x^2 - y^2}{2xy}$                       (c) 1                      (d)  $\frac{x^2 + y^2}{xy}$                       (e)  $\frac{x - y}{xy}$

17. If a right circular cylinder of height  $H$  and radius  $R$  were  $3/4$  full of water, how many times could a right circular cylinder of height  $H/2$  and radius  $R/2$  be filled with the water from the large cylinder?

- (a) 2                      (b) 4                      (c) 6                      (d) 8                      (e) none of these

18. Quadrilateral  $ABCD$  is a parallelogram with area 120. Point  $X$  cuts  $BC$  so that  $BX : XC = 3 : 2$ . Point  $Y$  cuts  $CD$  so that  $CY : YD = 2 : 1$ , and  $Z$  cuts  $AD$  so that  $AZ : ZD = 3 : 1$ . What is the area of the pentagon  $AXCYZ$ ?

- (a) 41                      (b) 47                      (c) 63                      (d) 73                      (e) 79



19. Read the following 5 statements carefully:

- (i) Statement (ii) is true.
- (ii) At most one of these 5 statements is true.
- (iii) All 5 of these statements are true.
- (iv)
- (v)

The last two statements are printed in invisible ink. Which of the statements are true?

- (a) only (i)    (b) only (iv) and (v)    (c) all of them    (d) none of them    (e) cannot be determined

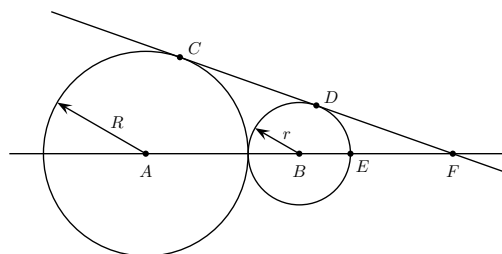


26. Sequence  $x_1, x_2, x_3, \dots$  satisfies  $x_1 + x_2 + \dots + x_n = n^3$  for all natural numbers  $n$ . A formula for  $x_n$  is:

(a)  $3n^3 - 3n^2 + 1$  (b)  $3n^2 + n + 1$  (c)  $3n^2 - 3n + 1$   
 (d)  $n^3 - 3n^2 + 3n - 1$  (e) none of these

27. Two circles are tangent to each other having radii of  $R$  and  $r$ , with  $r < R$ , and centered at  $A$  and  $B$ , respectively. If  $CD$  is a common tangent line of the circles (see picture), what is the length of  $EF$ ?

(a)  $\frac{2r^2}{R-r}$  (b)  $\frac{2R^2}{R-r}$  (c)  $\frac{r^2}{R-r}$   
 (d)  $\frac{R^2}{R-r}$  (e) none of these



28. If the perimeter of a rectangle is 40 feet, and the area of it  $75 \text{ ft}^2$ , how many feet long is the diagonal of the rectangle?

(a) 15 (b)  $\sqrt{15}$  (c)  $5\sqrt{10}$  (d)  $25\sqrt{10}$  (e) none of these

29. Which is true? The six-digit number  $abcabc$  is always divisible by

(a) 101 (b) 91 (c) 99 (d) 81 (e) 33

30. A rectangular picture has an area of  $120 \text{ in}^2$ . If the ratio of the length to the width is 6:5, what is the perimeter of the picture?

(a) 22 in (b) 42 in (c) 44 in (d) 52 in (e) none of these