

2005 STATE MATH CONTEST

GRADE 10-12 PRETEST

1. Eight swimmers competed in lanes 1-8 in a race. Those swimming in lanes 5, 6 and 7 finished in the last three places in some order. Three swimmers in adjacent lanes finished at the same place as their lane number. The other five swimmers finished in different places as their lane numbers, and only one of them had both numbers (lane and place) even, with the other four one of the numbers was even, the other was odd. Which lane did the winner swim on?
(a) lane 1 (b) lane 2 (c) lane 3 (d) lane 4 (e) lane 8

2. A circle of radius 10 is centered on the the x -axis, on the positive side of it, and is tangent to $y = \frac{4}{3}x$. The x -coordinate of the center of that circle is closest in value to which one of the following?
(a) 13 (b) 14 (c) 15 (d) 16 (e) 17

3. Which of the following functions satisfy $f(a + b) = f(a)f(b)$?
(i) $f(x) = 1$ (ii) $f(x) = 3^x$ (iii) $f(x) = |x|$ (iv) $f(x) = \sqrt{x}$
(a) only (ii) (b) only (ii) and (iv) (c) only (i) and (ii) (d) only (i) and (iv) (e) none of these

4. When $(x^{1/4} - x^{2/3})^7$ is multiplied out and simplified one of the terms has the form Kx^3 where K is a constant. Find K .
(a) 7 (b) -7 (c) 35 (d) -35 (e) none of these

5. Urn A contains 9 red balls and 11 white balls. Urn B contains 12 red balls and 3 white balls. One is to roll a single fair die. If the result is a one or a two, then one is to randomly select a ball from urn A. Otherwise one is to randomly select a ball from urn B. What is the probability of obtaining a red ball?

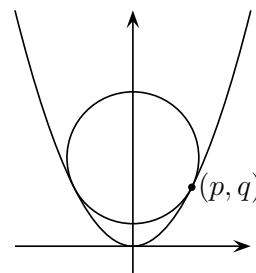
(a) $\frac{41}{60}$ (b) $\frac{19}{60}$ (c) $\frac{21}{35}$ (d) $\frac{35}{60}$ (e) none of these

6. What is the smallest perfect square number that is divisible by both 45 and 72?

(a) 360 (b) 1,600 (c) 3,240 (d) 3,600 (e) none of these

7. A circle of radius $4\sqrt{5}$ has its center on the y -axis and is tangent to the parabola $y = \frac{1}{8}x^2$. Point (p, q) is their common point in the first quadrant (see picture). What is the value of $p + q$?

(a) 14 (b) 15 (c) 16 (d) 17 (e) 18



8. A rectangular box having no top is to be twice as long as it is wide and is to hold 12 ft^3 . Which of the following is closest to the height (in feet) of the box of that type having the minimum surface area?

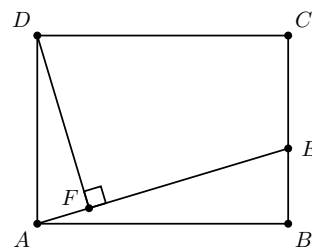
(a) $1/3$ (b) $1/2$ (c) $2/3$ (d) $3/4$ (e) 1

9. Two circles of radius 1 and 4 units are tangent to each other externally. The length of the part of the common tangent line between the points of tangency is closest to which of the following?

(a) 3.6 (b) 3.9 (c) 4.2 (d) 4.5 (e) 4.8

10. In rectangle $ABCD$ (see figure) side AB is one third longer than side BC . Point E divides BC such that EC is twice as long as BE . Point F is chosen on AE to make AE perpendicular to DF . Find the ratio of the area of triangle ADF to the area of rectangle $ABCD$.

(a) $4/41$ (b) $2/21$ (c) $3/34$ (d) $5/48$ (e) none of these



11. One root of $mx^2 - 10x + 3 = 0$ is two thirds of the other root. What is the sum of the roots?
- (a) $3/2$ (b) $5/2$ (c) $7/2$ (d) $1/4$ (e) $5/4$
12. Given that the fourth term of a geometric sequence is 108 and the 7th term is 2916, what is the first term of the sequence?
- (a) 3 (b) 4 (c) 8 (d) 12 (e) none of these
13. What is $\sin(\tan^{-1}(-4/3))$ equal to?
- (a) $-4/5$ (b) $4/3$ (c) $3/5$ (d) $-3/4$ (e) none of these
14. Which of the following are true for all $x > 0$, $y > 0$, $b > 0$, $b \neq 1$, and all positive integers n ?
- (i) $(x + y)(x - y) = x^2 - y^2$ (ii) $\sqrt{x + y} = \sqrt{x} + \sqrt{y}$ (iii) $\log_b(x^n) = n \log_b x$
 (iv) $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ (v) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
 (vi) $\log_b(x + y) = \log_b(x) + \log_b(y)$ (vii) $n^{x+y} = n^x n^y$
- (a) only (i), (iii), (iv) and (vii) are true (b) only (i), (iv) and (vii) are true
 (c) all are true (d) all except (ii) and (vi) are true (e) all except (ii) are true
15. A force of magnitude 15 lbs makes an angle of 60° with a force of magnitude 8 lbs. Which one of the following is closest to the magnitude of the resultant of those two forces?
- (a) 18 lbs (b) 20 lbs (c) 22 lbs (d) 24 lbs (e) 26 lbs